

1. Let \underline{c} be a constant, $f(x)$ be a function, and $P(x, y), Q(x)$ be predicates.
 - (a) Consider the sentence " $\forall x (P(x, \underline{c}) \rightarrow Q(f(x)))$ ". Show that this sentence is satisfiable (find a model for the sentence). Is this sentence logically valid? Prove your answer.
 - (b) Write a sentence (i.e. no free variables) involving $\underline{c}, f(x), P(x, y)$, and $Q(x)$ which is unsatisfiable. Explain why your sentence is unsatisfiable.
 - (c) Write a sentence (i.e. no free variables) involving $\underline{c}, f(x), P(x, y)$, and $Q(x)$ which is satisfiable but not logically valid. Construct models showing that your sentence is satisfiable but not logically valid.
 - (d) Write a sentence (i.e. no free variables) involving $\underline{c}, f(x), P(x, y)$, and $Q(x)$ which is logically valid. Prove that your sentence is logically valid.

Note: For parts (c), (d), and (e), don't reuse the sentence from part (a).

2. Prove Theorems K9, K10, K19, K25, K30, and K35