3.4 #16

3.5 #2

4.2 Use Cayley's theorem to write down a subgroup of S_8 which is isomorphic to the quaternion group $Q = \{\pm 1, \pm i, \pm j, \pm k\}$.

4.4 Consider the dihedral group $D_4 = \{1, x, x^2, x^3, y, xy, x^2y, x^3y\}$ where $x^4 = 1$, $y^2 = 1$, and $xy = yx^3$ (the rest of the relations follow from these). Given that the subgroups of D_4 are...

- $H_1 = \{1\}$
- $H_2 = \{1, x^2\}$
- $H_3 = \{1, y\}$
- $H_4 = \{1, xy\}$
- $H_5 = \{1, x^2y\}$
- $H_6 = \{1, x^3y\}$
- $H_7 = \{1, x, x^2, x^3\}$
- $H_8 = \{1, x^2, y, x^2y\}$
- $H_9 = \{1, x^2, xy, x^3y\}$
- $H_{10} = D_4$

Determine which subgroups of D_4 are normal.