Math 3110 "Group" Work

Split up into small groups of 3 or 4 people. Try to pick the same old people you always talk to. Don't try new things or meet new people. Anyone in class you don't know all ready must be weird (who knows they might be some sort of evil alien) so you shouldn't talk to them.

## Questions:

- Do the positive rational numbers,  $\mathbb{Q}_{>0}$ , form a group under multiplication? Why or why not?
- Do negative rational numbers,  $\mathbb{Q}_{<0}$ , form a group under multiplication? Why or why not?
- Does the half-open interval  $I = (0,1] = \{r \in \mathbb{R} \mid 0 < r \leq 1\}$  form a group under addition? How about multiplication?
- Make addition and multiplication tables for  $\mathbb{Z}_6$ .
- What is  $5^{-1}$  in  $\mathbb{Z}_6$ ? How about  $3^{-1}$ ?
- Does  $4^{-1}$  exist in  $\mathbb{Z}_{50}$ ? What about in  $\mathbb{Z}_{51}$ ? If the inverse exists, how do you compute it? Go ahead and do so if possible.
- Show that  $G = \left\{ \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} \mid a, b, c \in \mathbb{R} \right\}$  forms a group under matrix multiplication.
- Is G an Abelian group?
- Let G be a group. Show that  $(ab)^{-1} = a^{-1}b^{-1}$  for all  $a, b \in G$  if and only if G is Abelian.