

1. Workin' mod 15.

- (a) Find the additive inverse of each element in \mathbb{Z}_{15} .
- (b) Find the multiplicative inverse (it is exists) of each element in \mathbb{Z}_{15} .
[If a multiplicative inverse fails to exist, write "DNE" (does not exist).]
- (c) Compute $2^{-3} \cdot (5 - 11) \cdot 14^{999} + 13 \pmod{15}$.

2. The Euclidean Algorithm

- (a) Use the Euclidean Algorithm to find the greatest common divisor (gcd) of 1202 and 42.
- (b) Use the (extended) Euclidean Algorithm to find the greatest common divisor of $a = 303$ and $b = 63$, say $d = \gcd(a, b)$. Then determine integers x and y such that $ax + by = d$.
- (c) Use the (extended) Euclidean Algorithm to find 67^{-1} in $U(12345)$.

3. [Gallian Chapter 0 #8] Let $d = \gcd(a, b)$. If $a = da'$ and $b = db'$, show that $\gcd(a', b') = 1$.4. [Gallian Chapter 0 #10] Let $d = \gcd(a, b)$ and $\ell = \text{lcm}(a, b)$ (lcm = least common multiple). Show that if c divides a and c divides b , then c divides d . Also, show that if c is a multiple of a and c is a multiple of b , then c is a multiple of ℓ .