**Math 3110-101** **Homework #3** **Due: Wed., Sept. $18^{\text{th}}$, 2013**

1. A function problem

   (a) Let $f : \mathbb{Z} \to \mathbb{Z}$ be defined by $f(x) = 3x^2 + 1$.

      i. Show $f$ is not 1-1.
      ii. Show $f$ is not onto.
      iii. Let $A = \{-1, 0, 1, 3, 13\}$. Find $f(A)$.
      iv. Let $A = \{-1, 0, 1, 3, 13\}$. Find $f^{-1}(A)$.

   (b) Let $g : X \to Y$. Prove that $g$ is onto if and only if $g^{-1}(B) \neq \phi$ (the inverse image of $B$ is non-empty) for all non-empty subsets of $Y$: $\phi \neq B \subset Y$.

   Recall that for $A \subseteq X$ and $B \subseteq Y$ ...

$$f(A) = \{f(x) \mid x \in A\} \subseteq Y \qquad \text{and} \qquad f^{-1}(B) = \{x \in X \mid f(x) \in B\} \subseteq X$$

2. Dihedral groups: generators and relations style. Recall that ...

$$D_5 = \langle x, y \mid x^5 = 1,\ y^2 = 1,\ \text{and } (xy)^2 = 1 \rangle = \{1, x, x^2, x^3, x^4, y, xy, x^2y, x^3y, x^4y\}$$

   (a) Write down the Cayley table for $D_5$.

   (b) Find the inverse of each element (i.e. $1^{-1}$ =???, $x^{-1}$ =???, etc.).

   (c) Find the order of each element.

   (d) Find all of the **distinct** cyclic subgroups of $D_5$.

   (e) What is in $Z(D_5)$?

3. The Matrix problem

   (a) Compute $A^{-1}B^2$ where $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \in \mathrm{GL}_2(\mathbb{Z}_9)$

   (b) Find the cyclic subgroup generated by $A$. What is the order of $A$?

4. [Gallian Chapter 3 #42] Let $H$ be a subgroup of $G$ (a group). The set $C(H) = \{x \in G\ xh = hx \text{ for all } h \in H\}$ is called the *centralizer* of $H$. Prove that $C(H)$ is a subgroup of $G$.