Homework #4

Due: Wed., Oct. 9th, 2013

- 1. Orders of elements and number of such elements.
 - (a) Make a table which lists the possible orders of elements of \mathbb{Z}_{100} . List of the number such elements in the second row. [I'll get you started: There is 1 element of order 1 \odot] How many generators does \mathbb{Z}_{100} have?
 - (b) Repeat part (a) for D_{100} .
 - (c) How many elements of order 6 are there in $\mathbb{Z}_{74,070}$? What are they?
 - (d) How many elements of order 16 are there in $\mathbb{Z}_{74.070}$?
- 2. Let $g \in G$ (for some group G). Suppose |g| = 45. List the distinct elements of $\langle g^{18} \rangle$. Is $g^{102} \in \langle g^{18} \rangle$?
- 3. Let G be a finite group. Suppose that |a| = k and $|b| = \ell$ are relatively prime. Show that $\langle a \rangle \cap \langle b \rangle = \{e\}$.
- 4. Let G be a finite group and $a, b \in G$.
 - (a) Give a **concrete** example of a group G and elements a and b such that $|ab| \neq |a| \cdot |b|$.
 - (b) Prove (in general) that |ab| = |ba|.
- 5. For each of the following permutations:
 - i. Write the permutation as a product of disjoint cycles.
 - ii. Find its inverse.
 - iii. Find its order.
 - iv. Write it as a product of transpositions and state whether it is even or odd.
 - v. Conjugate it by $\sigma = (1526)$ (i.e. compute $\sigma \tau \sigma^{-1}$).
 - vi. Compute τ^{999} .
 - (a) $\tau = (1432)(56)(254)$
 - (b) $\tau = (1234)(1423)(246)$
 - (c) $\tau = (12)(345)(1357)$
- 6. Orders in S_n .
 - (a) What are the orders of the elements in S_5 ? Give an example of an element with each order.
 - (b) Does S_7 have an element of order 10? If so, find one. If not, explain why not.
 - (c) Does S_7 have an element of order 9? If so, find one. If not, explain why not.