## Homework #2

Due: Wed., Sept.  $9^{th}$ , 2015

- 1. Workin' mod 14.
  - (a) Find the additive inverse and order of each element in  $\mathbb{Z}_{14}$ .
  - (b) Find the multiplicative inverse or indicate "DNE" (does not exist) for each element in  $\mathbb{Z}_{14}$ . If the multiplicative inverse exists, that element belongs to U(14). In this case, find the order of that element (in U(14)).
  - (c) Compute  $5^{-2} \cdot (4-10) \cdot 13^{999} + 11 \pmod{14}$
  - (d) Compute  $A^{-1}$  given  $A = \begin{bmatrix} 1 & 5 \\ 4 & 9 \end{bmatrix} \in \mathrm{GL}_2(\mathbb{Z}_{14}).$
- 2. The Euclidean Algorithm
  - (a) Use the Euclidean Algorithm to find the greatest common divisor (gcd) of 1234 and 542.
  - (b) Use the (extended) Euclidean Algorithm to find the greatest common divisor of a = 1001 and b = 53, say  $d = \gcd(a, b)$ . Then determine integers x and y such that ax + by = d.
  - (c) Use the (extended) Euclidean Algorithm to find  $9^{-1}$  in U(1000).
- 3. Let  $d = \gcd(a, b)$ . If a = da' and b = db', show that  $\gcd(a', b') = 1$ . [Of course,  $a, a', b, b', d \in \mathbb{Z}$ .]
- 4. Show that for every  $n \in \mathbb{Z}$  we have  $n^3 \equiv n \pmod{6}$ .