

1. A function problem

- (a) Let $f : \mathbb{Z} \rightarrow \mathbb{Z}$ be defined by $f(x) = 2x^2 - 3$.
- Show f is not 1-1.
 - Show f is not onto.
 - Let $A = \{-1, 0, 1, 2, 3\}$. Find $f(A) = \{f(x) \mid x \in A\}$ (the image of the set A under the map f).
 - Let $A = \{-1, 0, 1, 2, 3\}$. Find $f^{-1}(A) = \{x \in \mathbb{Z} \mid f(x) \in A\}$ (the inverse image of A).
- (b) Let $g : X \rightarrow Y$. Prove that g is onto if and only if $g^{-1}(B) \neq \emptyset$ (the inverse image of B is non-empty) for all non-empty subsets of Y : $\emptyset \neq B \subset Y$.

Recall that for $A \subseteq X$ and $B \subseteq Y \dots$

$$f(A) = \{f(x) \mid x \in A\} \subseteq Y \quad \text{and} \quad f^{-1}(B) = \{x \in X \mid f(x) \in B\} \subseteq X$$

2. Dihedral groups: generators and relations style. Recall that ...

$$D_4 = \langle x, y \mid x^4 = 1, y^2 = 1, \text{ and } (xy)^2 = 1 \rangle = \{1, x, x^2, x^3, y, xy, x^2y, x^3y\}$$

- Write down the Cayley table for D_4 .
- Find the inverse of each element (i.e. $1^{-1} = ???$, $x^{-1} = ???$, etc.).
- Find the order of each element.
- Find all of the **distinct** cyclic subgroups of D_4 .
- What is in $Z(D_4)$ (recall that $Z(G)$ is the *center* of G)?
- Simplify $x^6y^{-3}x^3y^8x^{-5}yxy$.

3. The Matrix problem

- Compute $A^{-1}B^2$ where $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \in \text{GL}_2(\mathbb{Z}_9)$
- Find the cyclic subgroup generated by A . What is the order of A ?

4. Let H and K be subgroups of a group G . Show that $H \cap K$ (the intersection of H and K) is a subgroup of G .