Homework #4

Due: Wed., Sept. 30th, 2015

- 1. Orders of elements and number of such elements.
 - (a) Make a table which lists the possible orders of elements of \mathbb{Z}_{294} . List of the number such elements in the second row. [I'll get you started: There is 1 element of order 1 \odot] How many generators does \mathbb{Z}_{294} have?
 - (b) Repeat part (a) for D_{294} .
 - (c) How many elements of order 8 are there in $\mathbb{Z}_{1440000}$? What are they?
 - (d) How many elements of order 7 are there in $\mathbb{Z}_{1440000}$?
- 2. Let $g \in G$ (for some group G). Suppose |g| = 120. List the distinct elements of $\langle g^{100} \rangle$. Is $g^{30} \in \langle g^{100} \rangle$?
- 3. Let $g, x \in G$ (for some group G).
 - i. Show that $|x| = |gxg^{-1}|$ (i.e. conjugates have the same order).
 - ii. Prove or give a counterexample: $\langle x \rangle = \langle gxg^{-1} \rangle$.
- 4. For each of the following permutations:
 - i. Write the permutation as a product of disjoint cycles.
 - ii. Find its inverse.
 - iii. Find its order.
 - iv. Write it as a product of transpositions and state whether it is even or odd.
 - v. Conjugate it by $\sigma = (123)(45)$ (i.e. compute $\sigma \tau \sigma^{-1}$).
 - vi. Compute τ^{99} .
 - (a) $\tau = (124)(35)(24)(132)$
 - (b) $\tau = (1253)(354)(135)$
 - (c) $\tau = (12435)(134)(45)$
- 5. Orders in S_n .
 - (a) What are the orders of the elements in S_5 ? Give an example of an element with each order.
 - (b) Does S_{11} have an element of order 24? If so, find one. If not, explain why not.
 - (c) Does S_{11} have an element of order 16? If so, find one. If not, explain why not.