Math 3110-101 | # Homework #5 | **Due: Wed., Oct. 7th, 2015**

1. Cayley's theorem tells us that $Q = \{\pm 1, \pm i, \pm j, \pm k\}$ is isomorphic to a subgroup of S_8. Find such a subgroup using the ordering of elements: $1, -1, i, -i, j, -j, k, -k$ (so, for example, i is element #3). To help you get started, left multiplication by -1 sends 1 to -1, -1 to 1, i to $-i$, etc. so it sends 1 to 2, 2 to 1, 3 to 4, etc. Thus -1 corresponds with $(12)(34)(56)(78)$.

2. The following pairs of groups are isomorphic. Prove it.

   (a) $U(7)$ and $\mathbb{Z}_6$

   (b) $H = \left\{ \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} \;\middle|\; n \in \mathbb{Q} \right\}$ and $\mathbb{Q}$

3. The following pairs of groups are **<u>not</u>** isomorphic. Prove it.

   (a) $(\mathbb{Z}_5)^{2\times 2}$ and $\mathrm{GL}_2(\mathbb{R})$

   (b) $\mathbb{Z}_{222}$ and D_{111}

   (c) A_4 and D_6

   (d) $\mathbb{R}_{\neq 0}$ and $\mathbb{C}_{\neq 0}$ (non-zero reals and complex numbers both under multiplication)

4. Let $\varphi : G_1 \to G_2$ be an isomorphism

   (a) Show $\varphi(\langle g \rangle) = \langle \varphi(g) \rangle$ for any $g \in G_1$. [This implies that G_1 is cyclic iff G_2 is cyclic.]

   (b) Let K be a subgroup of G_2. Show that $\varphi^{-1}(K) = \{g \in G_1 \mid \varphi(g) \in K\}$ is a subgroup of G_1.