Math 3110-101 # Homework #6 **Due: Mon., Oct. 26^{th}, 2015**

1. Let H and K be subgroups of G.
   (a) Suppose that H and K are normal subgroups of G. Show that $H \cap K$ is a normal subgroup of G as well.
   (b) Let $|G| = 36$, $|H| = 12$, and $|K| = 18$. Using Lagrange's Theorem, what are the possible orders of $H \cap K$?
2. Let $H = \{1, x^3, x^6\} \subseteq D_9 = \{1, x, \ldots, x^8, y, xy, \ldots, x^8y\} = \langle x, y \mid x^9 = 1, y^2 = 1, xyxy = 1\rangle$. Notice $H = \langle x^3 \rangle$, so H is a subgroup of D_9. Quickly compute $[D_9 : H]$ (i.e. the index of H in D_9). Then find all of the left and right cosets of H in D_9. Is H a normal subgroup of D_9?
3. Direct products of cyclic groups.
   (a) Find the order of $(5, 2, 6)$ in $\mathbb{Z}_6 \oplus \mathbb{Z}_8 \oplus \mathbb{Z}_{99}$.
   (b) Explain why $\mathbb{Z}_{20} \cong \mathbb{Z}_4 \oplus \mathbb{Z}_5$ but $\mathbb{Z}_{20} \ncong \mathbb{Z}_2 \oplus \mathbb{Z}_{10}$.
4. Let G and H be groups.
   (a) Show $\{e\} \oplus H = \{(e, h) \mid h \in H\}$ is a normal subgroup of $G \oplus H$ (where e is the identity of G).
   Note: You need to show that $\{e\} \oplus H$ is a subgroup AND that it's normal.
   (b) Show $G \oplus H \cong H \oplus G$.