

1. Recall that the center of a group is a normal subgroup.  $Z = Z(D_6) = \{1, x^3\}$  where  $D_6 = \{1, x, \dots, x^5, y, xy, \dots, x^5y\}$ . Find the distinct cosets of  $Z$  in  $D_6$  then write down a Cayley table for  $D_6/Z$ . Is  $D_6/Z$  abelian? Is it cyclic? Explain your answer.

2. Let  $\varphi : D_n \rightarrow \{\pm 1\}$  be defined by  $\varphi(x) = \begin{cases} +1 & x \text{ is a rotation} \\ -1 & x \text{ is a reflection} \end{cases}$

Show that  $\varphi$  is a homomorphism. What is the kernel of  $\varphi$ ? What does the first isomorphism theorem tell us here?

3. Quotients in  $\mathbb{Z}_n$ .

(a) Let  $H = \langle 5 \rangle \subseteq \mathbb{Z}_{100}$ . First  $H = \{???\}$ . Then compute the cosets of  $H$  in  $\mathbb{Z}_{100}$ . Write down a Cayley table for  $\mathbb{Z}_{100}/H$ . What familiar group is this isomorphic to?

(b) Let  $k, \ell, n$  be positive integers such that  $n = k\ell$  (i.e.  $k$  divides  $n$ ). Make a conjecture about what the quotient  $\mathbb{Z}_n/\langle k \rangle$  is isomorphic to. Then prove your conjecture.

*Hint:* Define the map  $\varphi(x) = x$  from  $\mathbb{Z}_n$  to your target group and then use the first isomorphism theorem. Don't forget to show that  $\varphi$  is a well-defined homomorphism.

4. Let  $G$  be a finite group,  $H$  a normal subgroup of  $G$ , and  $g \in G$ . Show that  $|gH|$  (in the quotient group  $G/H$ ) divides  $|g|$  (in  $G$ ).