Math 3110-101 # Homework #1 **Due: Wed., Aug. 26^{th}, 2020**

1. Determine which of the following sets with operations are groups. If it is a group, state its identity, what the inverse of a typical element looks like, and determine if it is Abelian. If it is not a group, state which axioms hold and give counter-examples for those which fail (don't forget closure).

   (a) $(\mathbb{Z}_{\geq 0}, +)$ non-negative integers with addition

   (b) $(3\mathbb{Z}, +)$ multiples of 3 (i.e. $0, \pm 3, \pm 6, \ldots$) with addition

   (c) $(\mathbb{R}_{<0}, \cdot)$ negative reals with multiplication

   (d) $(\mathbb{R}_{\neq 0}, \div)$ non-zero reals with division

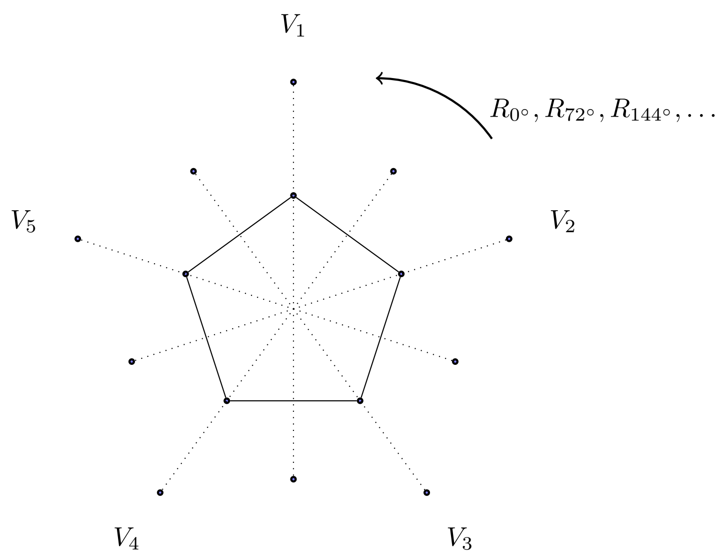
   (e) $(\mathbb{Q}_{>0}, \cdot)$ positive rationals with multiplication

2. Let G be a group with identity $e \in G$. Suppose that $g^2 = e$ for all $g \in G$.

   (a) What can be said about inverses of elements in G? Orders of elements?

   (b) Prove that G must be abelian.

3. Consider the dihedral group $D_5 = \{R_{0^\circ}, R_{72^\circ}, R_{144^\circ}, R_{216^\circ}, R_{288^\circ}, V_1, V_2, V_3, V_4, V_5\}$ (symmetries of a regular pentagon). [Rotations are done counter-clockwise and reflections are labeled in the picture below.]



   (a) Compute $V_1 R_{72^\circ}$, $R_{144^\circ} V_3$, and $V_2 V_5$.

   (b) Is D_5 Abelian? Why or why not?

   (c) Find the inverse of each element ($R_{0^\circ}^{-1} =???$, $R_{72^\circ}^{-1} =???$, etc.).

   (d) Find the order of each element.