Math 3110-101 # Homework #2 **Due: Wed., Sept. 2^{nd}, 2020**

1. Workin' mod 14.

   (a) Find the additive inverse and order of each element in $\mathbb{Z}_{14}$.

   (b) Find the multiplicative inverse or indicate "DNE" (does not exist) for each element in $\mathbb{Z}_{14}$. If the multiplicative inverse exists, that element belongs to $U(14)$. In this case, find the order of that element (in $U(14)$).

   (c) Compute $\quad 5^{-2} \cdot (4 - 10) \cdot 13^{999} + 11 \qquad \pmod{14}$.

   (d) Compute A^{-1} given $A = \begin{bmatrix} 1 & 5 \\ 4 & 9 \end{bmatrix} \in \mathrm{GL}_2(\mathbb{Z}_{14})$.

2. The Euclidean Algorithm

   (a) Use the Euclidean Algorithm to find the greatest common divisor (gcd) of 1234 and 542.

   (b) Use the (extended) Euclidean Algorithm to find the greatest common divisor of $a = 1001$ and $b = 53$, say $d = \gcd(a, b)$. Then determine integers x and y such that $ax + by = d$.

   (c) Use the (extended) Euclidean Algorithm to find 9^{-1} in $U(1000)$.

3. Let $d = \gcd(a, b)$. If $a = da'$ and $b = db'$, show that $\gcd(a', b') = 1$. [Of course, $a, a', b, b', d \in \mathbb{Z}$.]

4. Show that for every $n \in \mathbb{Z}$ we have $n^3 \equiv n \pmod{6}$.