Math 3110-101 | # Homework #4 | **Due: Wed., Sept. 30^{th}, 2020**

1. Orders of elements and number of such elements.

   (a) Make a table which lists the possible orders of elements of $\mathbb{Z}_{294}$. List of the number such elements in the second row. [I'll get you started: There is 1 element of order 1 ☺] How many generators does $\mathbb{Z}_{294}$ have?

   (b) Repeat part (a) for D_{294} [Does D_{294} have a generator? What is/are they or why not?]

   (c) How many elements of order 8 are there in $\mathbb{Z}_{1440000}$? What is/are they? or Why are there none?

   (d) How many elements of order 7 are there in $\mathbb{Z}_{1440000}$? What is/are they? or Why are there none?

2. Let $g \in G$ (for some group G). Suppose $|g| = 120$. List the *distinct* elements of $\langle g^{100} \rangle$. Is $g^{30} \in \langle g^{100} \rangle$?

3. Let $g, x \in G$ (for some group G).

   i. Show that $|x| = |gxg^{-1}|$ (i.e. conjugates have the same order).

   ii. Prove or give a counterexample: $\langle x \rangle = \langle gxg^{-1} \rangle$.

4. For each of the following permutations:

   i. Write the permutation as a product of disjoint cycles.

   ii. Find its inverse.

   iii. Find its order.

   iv. Write it as a product of transpositions and state whether it is even or odd.

   v. Conjugate it by $\sigma = (123)(45)$ (i.e. compute $\sigma\tau\sigma^{-1}$).

   vi. Compute τ^{99}.

   (a) $\tau = (124)(35)(24)(132)$

   (b) $\tau = (1253)(354)(135)$

   (c) $\tau = (12435)(134)(45)$

5. Orders in S_n.

   (a) What are the orders of the elements in S_5? Give an example of an element with each order.

   (b) Does S_{11} have an element of order 24? If so, find one. If not, explain why not.

   (c) Does S_{11} have an element of order 16? If so, find one. If not, explain why not.