Math 3110-101 **Homework #5** **Due: Fri., Oct. 6^{th}, 2023**

Please remember when submitting any work via email or in person to...

PUT YOUR NAME ON YOUR WORK!

#1 Let $\sigma = (123)(45)$. Fill out the following table:

$\tau =$	$(1432)(56)(254)$	$(1234)(1423)(246)$	$(12)(345)(1357)$
τ simplified (as disjoint cycles):			
The order of τ: $\lvert\tau\rvert =$			
The inverse of τ: $\tau^{-1} =$			
τ as a product of transpositions:			
τ conjugated by σ: $\sigma\tau\sigma^{-1} =$			
A power of τ: $\tau^{99} =$			

#2 Orders in S_n.

(a) What are the orders of the elements in S_7? Give an example of an element with each order.

Note: It might be helpful to know that there are 15 partitions of the number 7. Each partition corresponds to a different cycle type. Specifically, $7 = 6+1 = 5+2 = 5+1+1 = 4+3 = 4+2+1 = 4+1+1+1 = 3+3+1 = 3+2+2 = 3+2+1+1 = 3+1+1+1+1 = 2+2+2+1 = 2+2+1+1+1 = 2+1+1+1+1+1 = 1+1+1+1+1+1+1$.

(b) Does S_{10} have an element of order 30? If so, find one. If not, explain why not.

(c) Does S_{10} have an element of order 25? If so, find one. If not, explain why not.

RESUBMIT Type up Homework #4 Problems #4 and its solution in LaTeX.

Let $x, y \in G$ (for some group G). If there exists some $g \in G$ such $gxg^{-1} = y$, we say x and y are *conjugates*.

(a) Let $y = gxg^{-1}$ for some $g \in G$. Show that $|x| = |y|$ (i.e. conjugates have the same order).
Note: You will need to know that $(gxg^{-1})^k = gx^kg^{-1}$ for every non-negative integer k. I let you get away with informal justifications on original submissions. This time I would like you to include a careful proof of this fact (use induction on k).

(b) Prove or give a counterexample: $\langle x \rangle = \langle gxg^{-1} \rangle$ (where $x, g \in G$).
In other words, is it true or not that conjugates generate the same cyclic subgroup?

When typing this problem up, please write carefully: Restate the problem. Write in complete sentences.