Math 3110 | # Homework #2 | **Due: Fri., Sept. 6th, 2024**

Please remember when submitting any work via email or in person to...

PUT YOUR NAME ON YOUR WORK!

#1 Workin' mod whatever.

(a) Working mod 12. Create (and fill) a table as follows

(if an entry is undefined, write "DNE" = does not exist):

element $x =$	0	1	2	3	4	5	6	7	8	9	10	11
additive inverse $-x =$	0											
additive order $\|x\| =$	1											
mutiplicative inverse $x^{-1} =$	DNE											
multiplicative order $\|x\| =$	DNE											

Note: Additive inverses and orders go with the group structure of $(\mathbb{Z}_{12}, + \bmod 12)$ whereas the multiplicative inverses and orders go with the group structure of $(U(12), \cdot \bmod 12)$.

(b) Compute $2^{100} + (13 - 26) \cdot 5^{-2} \pmod 8$.

(c) Let $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$.

Explain why A does not belong to $\mathrm{GL}_2(\mathbb{Z}_{12})$.

Then explain why A does belong to $\mathrm{GL}_2(\mathbb{Z}_7)$ and compute A^{-1}.

#2 Use the extended Euclidean Algorithm to find 109^{-1} in $U(4115)$.

Show your work. Simplify your answer (your answer should be a number between 0 and 4114).

#3 Let $a, b \in \mathbb{Z}$ (not both zero) such that $d = \gcd(a, b)$. In addition, let $a = da'$ and $b = db'$.

Show that $\gcd(a', b') = 1$.

#4 Show that for every $n \in \mathbb{Z}$, we have $n^3 = n \pmod 6$.

#5 RESUBMIT Type up Homework #1 Problem #2 and its solution in LaTeX.

Let G be a group with identity $e \in G$.

(a) Give a concrete example of a group G and elements $a, b \in G$ where $(ab)^{-1} \neq a^{-1}b^{-1}$.

(b) Prove G is an abelian group if and only if for all $a, b \in G$, $(ab)^{-1} = a^{-1}b^{-1}$.

When typing this problem up, write it up carefully: Restate the problem. Write in complete sentences.