Math 3110 # Homework #3 **Due: Fri., Sept. 13th, 2024**

Please remember when submitting any work via email or in person to...

PUT YOUR NAME ON YOUR WORK!

#1 A function problem. Recall that for a function $h : X \to Y$, if $A \subseteq X$ and $B \subseteq Y$, then...

$$h(A) = \{h(x) \mid x \in A\} \subseteq Y \qquad \text{and} \qquad h^{-1}(B) = \{x \in X \mid h(x) \in B\} \subseteq X$$

(a) Let $f : \mathbb{Z} \to \mathbb{Z}$ be defined by $f(x) = |x^2 - 1| - 3$.
Suggestion: Start off by making a table of values of $f(x)$ for small x's to see what outputs look like.

i. Explain why f is not 1-1.
ii. Explain why f is not onto.
iii. Let $A = \{-3, -2, -1, 0, 1, 2\}$. Find $f(A) = \{f(x) \mid x \in A\}$.
(This is the image of the set A under the map f.)
iv. Let $B = \{0, 1, 2, 3, 4, 5\}$. Find $f^{-1}(B) = \{x \in \mathbb{Z} \mid f(x) \in B\}$.
(This is the inverse image of B under the map f.)

(b) Let $g : X \to Y$.
For $A, B \subseteq X$ show $g(A \cup B) = g(A) \cup g(B)$.
Then for $C, D \subseteq Y$ show $g^{-1}(C \cap D) = g^{-1}(C) \cap g^{-1}(D)$.

#2 Dihedral groups: generators and relations style. Recall that ...

$$D_n = \langle x, y \mid x^n = 1,\ y^2 = 1,\ \text{and } (xy)^2 = 1 \rangle = \{1, x, x^2, \ldots, x^{n-1}, y, xy, x^2y, \ldots, x^{n-1}y\}$$

(a) Write down the Cayley table for D_4.

(b) Fill out the following table for D_7:

element $g =$	1	x	x^2	x^3	x^4	x^5	x^6	y	xy	x^2y	x^3y	x^4y	x^5y	x^6y
inverse $g^{-1} =$	1													
order $\lvert g \rvert =$	1													

(c) Simplify $y^{-8}x^{20}y^3x^{-3}y^{99}x^5$ in D_{12}.

#3 Let $H = \left\{ \begin{bmatrix} a & 2b \\ 0 & 1 \end{bmatrix} \,\middle|\, a, b \in \mathbb{Q} \text{ and } a \neq 0 \right\}$. Show H is a subgroup of $\mathrm{GL}_2(\mathbb{Q})$.

#4 Let G be an abelian group and $K = \{g \in G \mid g = g^{-1}\}$.

(a) Show K is a subgroup of G.
(b) Give an example of a *non-abelian* group G where K (defined the same way) is not a subgroup.

#5 RESUBMIT Type up Homework #2 Problem #3 and its solution in LaTeX.

Let $a, b \in \mathbb{Z}$ (not both zero) such that $d = \gcd(a, b)$. In addition, let $a = da'$ and $b = db'$.
Show that $\gcd(a', b') = 1$.

When typing this problem up, write it up carefully: Restate the problem. Write in complete sentences.