Math 3110 # Homework #4 **Due: ~~Fri., Sept. 27$^{\text{th}}$, 2024~~**

Please remember when submitting any work via email or in person to...

PUT YOUR NAME ON YOUR WORK!

#1 The Matrix... ...problem

(a) Compute $A^{-1}B^2$ where $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} \in \mathrm{GL}_2(\mathbb{Z}_{13})$

Hint: Not so random fact $(-5)13 + (6)11 = 1$.

(b) Find the cyclic subgroup generated by $B = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$ (in $\mathrm{GL}_2(\mathbb{Z}_{13})$). What is the order of B?

#2 Orders of elements and number of such elements.

(a) Make a table which lists the possible orders of elements of $\mathbb{Z}_{825}$. List the number such elements in the second row.

Order =	1	???	$\cdots$
Number of such elements =	1	???	$\cdots$

How many generators does $\mathbb{Z}_{825}$ have?

Suggestion: Type "divisors of 825" into a tool like WolframAlpha.

(b) Repeat part (a) for D_{242}

Order =	1	???	$\cdots$
Number of such elements =	1	???	$\cdots$

Does D_{242} have a generator? What is/are they? or Why not?

(c) How many elements of order 9 are there in $\mathbb{Z}_{12345654321}$? What is/are they? or Why are there none?

(d) How many elements of order 6 are there in $\mathbb{Z}_{12345654321}$? What is/are they? or Why are there none?

#3 Let $g \in G$ (for some group G). Suppose $|g| = 132$. List the *distinct* elements of $\langle g^{110} \rangle$.

#4 Let $x, y \in G$ (for some group G). If there exists some $g \in G$ such $gxg^{-1} = y$, we say x and y are *conjugates*.

(a) Use induction to show $(gxg^{-1})^k = gx^kg^{-1}$ for any $k \in \mathbb{Z}_{\geq 0}$.

Note: Consequently, $(gxg^{-1})^n = e$ iff $gx^ng^{-1} = e$ iff $x^n = g^{-1}eg = e$, so conjugates have the same order.

(b) Show xy and yx are conjugate.

(c) Putting (a) and (b) together, we have $|xy| = |yx|$ and so $\langle xy \rangle$ and $\langle yx \rangle$ have the same size.

Prove or give a counterexample: $\langle xy \rangle = \langle yx \rangle$ (where $x, y \in G$).