

Homework #5

Revision Problem

Due: ~~Fri., Oct. 4th~~, 2024Name: My Name Goes Here

Revision Problem (Homework #4 Problem #4(a)): Let $x, y \in G$ (for some group G). If there exists some $g \in G$ such $gxg^{-1} = y$, we say x and y are *conjugates*. Our problem is to use induction to show $(gxg^{-1})^k = gx^k g^{-1}$ for any $k \in \mathbb{Z}_{\geq 0}$.

THEOREM: Let G be a group and $g \in G$. For any $k \in \mathbb{Z}_{\geq 0}$, we have $(gxg^{-1})^k = gx^k g^{-1}$.

Proof: We use induction on k to prove the stated equality.

Base Case: This is my base case.

Inductive Step: This is my inductive step.

Therefore, by induction, $(gxg^{-1})^k = gx^k g^{-1}$ for all $k \in \mathbb{Z}_{\geq 0}$. \square

Note: As a consequence of the above theorem, $(gxg^{-1})^n = e$ if and only if $gx^n g^{-1} = e$ if and only if $x^n = g^{-1}eg = e$. Therefore, conjugate elements always have the same order.