The Cayley table for $A_4 = \{(1), (123), (132), (124), (142), (134), (143), (234), (243), (12)(34), (13)(24), (14)(23)\}$.

	(1)	(12)(34)	(13)(24)	(14)(23)	(123)	(243)	(142)	(134)	(132)	(143)	(234)	(124)
(1)	(1)	(12)(34)	(13)(24)	(14)(23)	(123)	(243)	(142)	(134)	(132)	(143)	(234)	(124)
(12)(34)	(12)(34)	(1)	(14)(23)	(13)(24)	(243)	(123)	(134)	(142)	(143)	(132)	(124)	(234)
(13)(24)	(13)(24)	(14)(23)	(1)	(12)(34)	(142)	(134)	(123)	(243)	(234)	(124)	(132)	(143)
(14)(23)	(14)(23)	(13)(24)	(12)(34)	(1)	(134)	(142)	(243)	(123)	(124)	(234)	(143)	(132)
(123)	(123)	(134)	(243)	(142)	(132)	(124)	(143)	(234)	(1)	(14)(23)	(12)(34)	(13)(24)
(243)	(243)	(142)	(123)	(134)	(143)	(234)	(132)	(124)	(12)(34)	(13)(24)	(1)	(14)(23)
(142)	(142)	(243)	(134)	(123)	(234)	(143)	(124)	(132)	(13)(24)	(12)(34)	(14)(23)	(1)
(134)	(134)	(123)	(142)	(243)	(124)	(132)	(234)	(143)	(14)(23)	(1)	(13)(24)	(12)(34)
(132)	(132)	(234)	(124)	(143)	(1)	(13)(24)	(14)(23)	(12)(34)	(123)	(142)	(134)	(243)
(143)	(143)	(124)	(234)	(132)	(12)(34)	(14)(23)	(13)(24)	(1)	(243)	(134)	(142)	(123)
(234)	(234)	(132)	(143)	(124)	(13)(24)	(1)	(12)(34)	(14)(23)	(142)	(123)	(243)	(134)
(124)	(124)	(143)	(132)	(234)	(14)(23)	(12)(34)	(1)	(13)(24)	(134)	(243)	(123)	(142)

Let's find all of the subgroups of A_4. First, we have all of the cyclic subgroups.

- $\langle(1)\rangle = \{(1)\}$
- $\langle(12)(34)\rangle = \{(1), (12)(34)\}$
- $\langle(13)(24)\rangle = \{(1), (13)(24)\}$
- $\langle(14)(23)\rangle = \{(1), (14)(23)\}$
- $\langle(123)\rangle = \langle(132)\rangle = \{(1), (123), (132)\}$
- $\langle(124)\rangle = \langle(142)\rangle = \{(1), (124), (142)\}$
- $\langle(134)\rangle = \langle(143)\rangle = \{(1), (134), (143)\}$
- $\langle(234)\rangle = \langle(243)\rangle = \{(1), (234), (243)\}$

By looking at the Cayley table we can see if we tried to form a subgroup with a couple of 3-cycles (which aren't inverses of each other), we end up generating all of A_4. For example: $(123)(134) = (234)$ so if a subgroup contains (123) and (134), it must also contain (234) and inverses and the identity — we're already up to $2 + 2 + 2 + 1 = 7$ elements and since the order of a subgroup divides $|A_4| = 12$, we must conclude that any subgroup containing both (123) and (134) is all of A_4.

Next, what if we try to have a 3-cycle and an element like (12)(34)? Say (123) and (12)(34). Well, $(123)(12)(34) = (134)$ so we have two different 3-cycles again and thus we must generate the whole group A_4.

Summing up (so far), any subgroup with at least one 3-cycle must either be one of the 4 cyclic subgroups of order 3 or all of A_4.

What about the non 3-cycle elements? $(12)(34)\ (13)(24) = (14)(23)$ So to we can't have a subgroup with just two of these elements. We must include all 3 of them. Let's look at $H = \{(1), (12)(34), (13)(24), (14)(23)\}$. Looking at the table, we can see H is closed – so H is a subgroup (by the finite subgroup test).

Therefore, adding

- $H = \{(1), (12)(34), (13)(24), (14)(23)\}$
- A_4

to the list (of cyclic subgroups) completes our list of all of the subgroups of A_4. Our next question is, "Which of these subgroups are normal?"

The following calculations show that $gKg^{-1} \neq K$ for each cyclic subgroup K (other than the trivial subgroup $\langle(1)\rangle$). Therefore, they are **not** normal. (Does that mean they're *weird*?)

- $(123)\langle(12)(34)\rangle(123)^{-1} = \langle(14)(23)\rangle \neq \langle(12)(34)\rangle$
- $(123)\langle(13)(24)\rangle(123)^{-1} = \langle(12)(34)\rangle \neq \langle(13)(24)\rangle$
- $(123)\langle(14)(23)\rangle(123)^{-1} = \langle(13)(24)\rangle \neq \langle(14)(23)\rangle$
- $(124)\langle(123)\rangle(124)^{-1} = \langle(243)\rangle \neq \langle(123)\rangle$
- $(123)\langle(124)\rangle(123)^{-1} = \langle(234)\rangle \neq \langle(124)\rangle$
- $(123)\langle(134)\rangle(123)^{-1} = \langle(142)\rangle \neq \langle(134)\rangle$
- $(123)\langle(234)\rangle(123)^{-1} = \langle(143)\rangle \neq \langle(234)\rangle$

Of course the trivial subgroup $\langle(1)\rangle$ and A_4 itself are normal in A_4. (For any group G, {e} and G are normal subgroups of G.)

Finally, consider $H = \{(1), (12)(34), (13)(24), (14)(23)\}$.

- $H = \{(1), (12)(34), (13)(24), (14)(23)\}$
  $H = (1)H = (12)(34)H = (13)(24)H = (14)(23)H = H(1) = H(12)(34) = H(13)(24) = H(14)(23)$
- $(123)H = \{(123), (134), (243), (142)\}$
  $(123)H = (134)H = (243)H = (142)H = H(123) = H(134) = H(243) = H(142)$
- $(132)H = \{(132), (234), (124), (143)\}$
  $(132)H = (234)H = (124)H = (143)H = H(132) = H(234) = H(124) = H(143)$

So $H \lhd A_4$.

Let's look at all of the quotients of A_4. First, the trivial cases.

- $A_4 / A_4 \cong \{1\}$ $\quad$ $[A_4 : A_4] = 1$ so the quotient group has order 1 and thus is the trivial group.
- $A_4 / \{(1)\} \cong A_4$ $\quad$ Quotienting by the trivial subgroup "does nothing".

The quotient by $H = \{(1), (12)(34), (13)(24), (14)(23)\}$ is more interesting.

$$\left| A_4 / H \right| = |A_4| / |H| = [A_4 : H] = \frac{12}{4} = 3$$

Since there is only 1 group order 3 (up to isomorphism), $A_4 / H \cong \mathbb{Z}_3$

Example: Multiplying cosets.

$$(243)H\ (124)H = (243)(124)H = (14)(23)H = H$$

Therefore, $((243)H)^{-1} = (124)H$.

Working out all of the other cases, we get the following Cayley table for A_4 / H:

	H	(123)H	(132)H
H	H	(123)H	(132)H
(123)H	(123)H	(132)H	H
(132)H	(132)H	H	(123)H

$\cong$

	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

Notice that since we ordered the elements in the original Cayley table according to cosets of H, we have 4×4 "blocks" of the original table corresponding to the entries of the quotient group's table.