You should know...

- the definition of a homomorphism, an isomorphism, and an automorphism.
- the definition of the kernel of a homomorphism.
- if $\varphi : G \to G'$ is a homomorphism, $\text{Ker}(\varphi)$ is a normal subgroup of G, $\varphi(G)$ is a subgroup of G'.
- homomorphisms send elements of order k to elements of order ℓ where ℓ divides k.
- isomorphisms send elements of order k to elements of order k.
- how to write a group as a group of permutations (via Cayley's theorem). Try for example: 4.2 #1.
- Lagrange's theorem! which implies that the order of a subgroup divides the order of the group and the order of an element divides the order of the group.
- the definition of a left/right coset.
- if H is a subgroup, the number of left cosets of H = the number of right cosets of H.
- if H is a subgroup, all cosets of H have the same size.
- the definition of a normal subgroup.
- the definition of the center of a group, $Z(G)$, and that $Z(G)$ is normal in G.
- we can only quotient by **normal** subgroups.
- how to multiply cosets in a quotient group.
- the first isomorphism theorem: if $\varphi : G \to G'$ is a homomorphism, then $G/\text{Ker}(\varphi) \cong \varphi(G)$. In words, "$G$ mod the kernel is isomorphic to the image."

Do your homework! Also, some sample suggested problems...

- 3.4: 2, 7, 14, 21
- 3.5: 2, 14
- 4.2: 1, 3, 6
- 4.4: 5, 9, 10, 35
- 4.5: 1, 3, 19, 22

Quick questions to get you thinking:

1. Is $\mathbb{Z}_5 \cong \mathbb{Z}$?
2. Is $D_5 \cong \mathbb{Z}_{10}$?
3. Is $D_4 \cong Q$ (the quaternion group)?
4. Is $\{\pm 1, \pm i\} \cong \mathbb{Z}_4$?
5. How many elements of order 7 does S_6 have?
6. $\varphi : A_4 \to \mathbb{Z}_{25}$ is a homomophism. What is φ?

Answers...

1. Is $\mathbb{Z}_5 \cong \mathbb{Z}$? No. Orders don't match! (5 vs. ∞)
2. Is $D_5 \cong \mathbb{Z}_{10}$? No. D_5 is not abelian but $\mathbb{Z}_{10}$ is. (there are lots of other reasons too...like not cyclic vs. cyclic...or...the orders of the elements don't match etc.)
3. Is $D_4 \cong Q$ (the quaternion group)? No. Q has 6 elements of order 4 (namely $\pm i, \pm j, \pm k$) whereas D_4 has only 2 elements of order 4 (the 90 and 270 degree rotations).
4. Is $\{\pm 1, \pm i\} \cong \mathbb{Z}_4$? Yes. They're both cyclic of order 4. We know that any two cyclic groups of the same order are isomorphic.
5. How many elements of order 7 does S_6 have? None. 7 does not divide $|S_6| = 6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$.
6. $\varphi : A_4 \to \mathbb{Z}_{25}$ is a homomophism. What is φ? The image of φ is a subgroup of $\mathbb{Z}_{25}$ so it's order must be either 1, 5, or 25 (divisors of 25). On the other hand, we know by the first isomorphism theorem, $|A_4/\text{Ker}(\varphi)| = |\varphi(A_4)|$ which means that $12 = |A_4| = |\text{Ker}(\varphi)| \cdot |\varphi(A_4)|$ so the order of the image of φ must divide 12 (the order of the domain) as well. Thus it's order is either 1, 2, 3, 4, 6, or 12. Therefore, the only possible order for the image is 1. Since the image is a subgroup, it contains the identity. Therefore, $\varphi(A_4) = \{0\}$. Thus $\varphi(\sigma) = 0$ for all $\sigma \in A_4$. Hmmm...that wasn't so "quick" afterall. Oh well.