Math 3110-101

Homework #1

Due: Fri., Jan. 23^{rd}, 2015

1. Determine which of the following sets with operations are groups. If it is a group, state its identity, what the inverse of a typical element looks like, and determine if it is Abelian. If it is not a group, state which axioms hold and give counter-examples for those which fail (don't forget closure).

   (a) $(\mathbb{Q}_{\geq 0}, +)$ non-negative rationals with addition

   (b) $(\mathbb{R}_{>0}, \cdot\,)$ positive reals with multiplication

   (c) $(\mathbb{Z}, -)$ integers with subtraction

   (d) $(5\mathbb{Z}, +)$ multiples of 5 (i.e. $0, \pm 5, \pm 10, \dots$) with addition

   (e) $(\mathbb{Q}_{<0}, \cdot\,)$ negative rationals with multiplication

2. Let G be a group. Show that G is Abelian if and only if $(ab)^2 = a^2b^2$ for all $a, b \in G$.

3. Create a Cayley table for $D_3 = \{R_0, R_{120}, R_{240}, D, D', V\}$ (symmetries of an equilateral triangle).

   Is D_3 Abelian?

   Find the inverse of each element ($R_0^{-1} =$???, $R_{120}^{-1} =$???, etc.).