Math 3110-101 **Homework #2** **Due: Fri., Jan. 30^{th}, 2015**

1. Workin' mod 15.

   (a) Find the additive inverse of each element in $\mathbb{Z}_{15}$.

   (b) Find the multiplicative inverse (it is exists) of each element in $\mathbb{Z}_{15}$.
   [If a multiplicative inverse fails to exist, write "DNE" (does not exist).]

   (c) Compute $2^{-3} \cdot (5-11) \cdot 14^{999} + 13 \pmod{15}$.

2. The Euclidean Algorithm

   (a) Use the Euclidean Algorithm to find the greatest common divisor (gcd) of 1202 and 42.

   (b) Use the (extended) Euclidean Algorithm to find the greatest common divisor of $a = 303$ and $b = 63$, say $d = \gcd(a, b)$. Then determine integers x and y such that $ax + by = d$.

   (c) Use the (extended) Euclidean Algorithm to find 67^{-1} in $U(12345)$.

3. [Gallian Chapter 0 #8] Let $d = \gcd(a, b)$. If $a = da'$ and $b = db'$, show that $\gcd(a', b') = 1$.

4. [Gallian Chapter 0 #10] Let $d = \gcd(a, b)$ and $\ell = \text{lcm}(a, b)$ (lcm = least common multiple). Show that if c divides a and c divides b, then c divides d. Also, show that if c is a multiple of a and c is a multiple of b, then c is a multiple of ℓ.