Math 3110-101 # Homework #4 **Due: Wed., Feb. 25^{th}, 2015**

1. Orders of elements and number of such elements.
   (a) Make a table which lists the possible orders of elements of $\mathbb{Z}_{100}$. List of the number such elements in the second row. [I'll get you started: There is 1 element of order 1 ☺] How many generators does $\mathbb{Z}_{100}$ have?
   (b) Repeat part (a) for D_{100}.
   (c) How many elements of order 6 are there in $\mathbb{Z}_{74,070}$? What are they?
   (d) How many elements of order 16 are there in $\mathbb{Z}_{74,070}$?
2. Let $g \in G$ (for some group G). Suppose $|g| = 45$. List the *distinct* elements of $\langle g^{18} \rangle$. Is $g^{102} \in \langle g^{18} \rangle$?
3. Let G be a finite group. Suppose that $|a| = k$ and $|b| = \ell$ are relatively prime. Show that $\langle a \rangle \cap \langle b \rangle = \{e\}$.
4. Let G be a finite group and $a, b \in G$.
   (a) Give a **concrete** example of a group G and elements a and b such that $|ab| \neq |a| \cdot |b|$.
   (b) Prove (in general) that $|ab| = |ba|$.
5. For each of the following permutations:
   i. Write the permutation as a product of disjoint cycles.
   ii. Find its inverse.
   iii. Find its order.
   iv. Write it as a product of transpositions and state whether it is even or odd.
   v. Conjugate it by $\sigma = (1526)$ (i.e. compute $\sigma\tau\sigma^{-1}$).
   vi. Compute τ^{999}.

   (a) $\tau = (1432)(56)(254)$
   (b) $\tau = (1234)(1423)(246)$
   (c) $\tau = (12)(345)(1357)$
6. Orders in S_n.
   (a) What are the orders of the elements in S_5? Give an example of an element with each order.
   (b) Does S_7 have an element of order 10? If so, find one. If not, explain why not.
   (c) Does S_7 have an element of order 9? If so, find one. If not, explain why not.