Math 3110-101 # Homework #5 **Due: Fri., Mar. 27^{th}, 2015**

1. Let H and K be subgroups of G.
   (a) Prove that $H \cap K$ is a subgroup of G.
   (b) Suppose that H and K are normal subgroups of G. Show that $H \cap K$ is a normal subgroup of G as well.
   (c) Let $|G| = 100$, $|H| = 50$, and $|K| = 20$. Using Lagrange's Theorem, what are the possible orders of $H \cap K$?
2. Let $H = \{1, x^2, x^4\} \subseteq D_6 = \{1, x, \ldots, x^5, y, xy, \ldots, x^5y\}$. Notice $H = \langle x^2 \rangle$, so H is a subgroup of D_6. Quickly compute $[D_6 : H]$ (i.e. the index of H in D_6). Then find all of the left and right cosets of H in D_6. Is H a normal subgroup of D_6?
3. Let $G = \langle a \rangle$ be some cyclic group and suppose that $|a| = 15$. Find all of the left cosets of $H = \langle a^5 \rangle$ in $G = \langle a \rangle$.
4. Explain why $\mathbb{Z}_{12} \cong \mathbb{Z}_3 \oplus \mathbb{Z}_4$ but $\mathbb{Z}_{12} \ncong \mathbb{Z}_2 \oplus \mathbb{Z}_6$.
5. Let G and H be groups.
   (a) Show $G \oplus H$ is abelian if and only if G and H are abelian.
   (b) Show $G \oplus H \cong H \oplus G$.