Math 3110-101 | # Homework #6 | **Due: Wed., Apr. 8^{th}, 2015**

1. Recall that the center of a group is a normal subgroup. $Z = Z(Q) = \{\pm 1\}$ where $Q = \{\pm 1, \pm i, \pm j, \pm k\}$ is the quaternion group. Find the distinct cosets of Z in Q then write down a Cayley table for Q/Z. Is Q/Z abelian? Is it cyclic? Explain your answer.

2. Let $\varphi : D_n \to \{\pm 1\}$ be defined by $\varphi(x) = \begin{cases} +1 & x \text{ is a rotation} \\ -1 & x \text{ is a reflection} \end{cases}$

   Show that φ is a homomorphism. What is the kernel of φ? What does the first isomorphism theorem tell us here?

3. Quotients in $\mathbb{Z}_n$.

   (a) Let $H = \langle 5 \rangle \subseteq \mathbb{Z}_{100}$. First $H = \{???\}$. Then compute the cosets of H in $\mathbb{Z}_{100}$. Write down a Cayley table for $\mathbb{Z}_{100}/H$. What familiar group is this isomorphic to?

   (b) Let k, ℓ, n be positive integers such that $n = k\ell$ (i.e. k divides n). Make a conjecture about what the quotient $\mathbb{Z}_n/\langle k \rangle$ is isomorphic to. Then prove your conjecture.

   Hint: Define the map $\varphi(x) = x$ from $\mathbb{Z}_n$ to your target group and then use the first isomorphism theorem. Don't forget to show that φ is a well-defined homomorphism.

4. Let G be a finite group, H a normal subgroup of G, and $g \in G$. Show that $|gH|$ (in the quotient group G/H) divides $|g|$ (in G).