Homework #1

Please remember when submitting any work via email or in person to...

> ## PUT YOUR NAME ON YOUR WORK!

#1. Group axiom basics.

(a) Explain why positive rationals with multiplication, $(\mathbb{Q}_{>0}, \cdot)$, is a group, but negative rationals with multiplication $(\mathbb{Q}_{<0}, \cdot)$ is not.

(b) Which group axioms hold and which fail if we consider $(\mathbb{R}_{\neq 0}, \div)$ non-zero reals with division? Give **concrete** counter-examples for axioms that fail.

(c) Consider the maximum operation: $x \star y = \max\{x, y\}$. For example, $-8 \star 5 = \max\{-8, 5\} = 5$ since 5 is greater than -8. Is $\mathbb{Z}$, the integers, equipped with the max operation a group? Which axioms hold? Give a proof if an axiom holds or a **concrete** counter-example if one fails. Also, is this operation commutative?

#2. Let G be a group with identity $e \in G$. Suppose that $g^2 = e$ for all $g \in G$.

(a) What can be said about inverses of elements in G? What can be said about orders of elements?

(b) Prove that G must be abelian.

#3. Consider the dihedral group $D_5 = \{R_{0^\circ}, R_{72^\circ}, R_{144^\circ}, R_{216^\circ}, R_{288^\circ}, V_1, V_2, V_3, V_4, V_5\}$ (symmetries of a regular pentagon). [Rotations are done counter-clockwise and reflections are labeled in the picture below.]
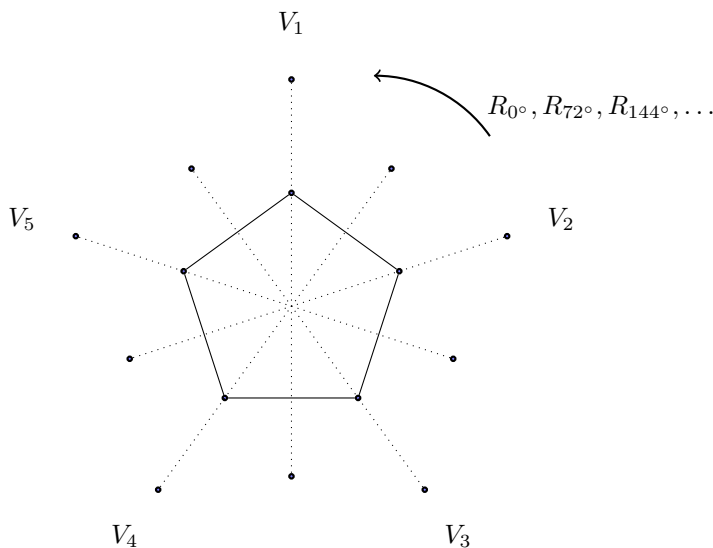
(a) Compute $V_1 R_{72^\circ}$, $R_{144^\circ} V_3$, and $V_2 V_5$. [Draw some pictures!]

(b) Is D_5 Abelian? Why or why not?

(c) Make a table of inverses and orders for each element:

Element:	$g =$	R_{0°	R_{72°	R_{144°	R_{216°	R_{288°	V_1	V_2	V_3	V_4	V_5
Inverse:	$g^{-1} =$	???	$\cdots$								
Order:	$\lvert g \rvert =$	???	$\cdots$								