Math 3110-101 **Homework #2** **Due: Fri., Feb. 5th, 2021**

Please remember when submitting any work via email or in person to...

PUT YOUR NAME ON YOUR WORK!

#1 Workin' mod 14.

(a) Fill out the following table (if an entry is undefined, write "DNE" = does not exist):

element $x =$	0	1	2	3	4	5	6	7	8	9	10	11	12	13
additive inverse $-x =$	0													
additive order $\|x\| =$	1													
mutiplicative inverse $x^{-1} =$	DNE													
multiplicative order $\|x\| =$	DNE													

Note: Additive inverses and orders go with the group structure of $(\mathbb{Z}_{14}, +$ mod 14$)$ whereas the multiplicative inverses and orders go with the group structure of $(U(14), \cdot$ mod 14$)$.

(b) Compute $3^{-5} \cdot (4 - 12) + 2^{123} \pmod{14}$.

(c) Let $A = \begin{bmatrix} 3 & 4 \\ 3 & 1 \end{bmatrix}$.

Explain why A does not belong to $\mathrm{GL}_2(\mathbb{Z}_6)$.

Then explain why A does belong to $\mathrm{GL}_2(\mathbb{Z}_{14})$ and compute A^{-1}.

#2 Use extended Euclidean Algorithm to find 15^{-1} in $U(1234)$.

Show your work. Simplify your answer (your answer should be a number between 0 and 1233).

#3 Let $a, b, n \in \mathbb{Z}$ and $n > 1$. Suppose that $\gcd(a, n) = 1$ and $\gcd(b, n) = 1$. Show that $\gcd(ab, n) = 1$.

#4 Let $a, b, x, y \in \mathbb{Z}$. Suppose that $ax + by = 10$.

What can be said about $\gcd(a, b)$?

If $\gcd(x, y) = 10$, what can be said about $\gcd(a, b)$?

RESUBMIT Type up Homework #1 Problem #2 and its solution in LaTeX.

Let G be a group with identity $e \in G$. Suppose that $g^2 = e$ for all $g \in G$.

(a) What can be said about inverses of elements in G? What can be said about orders of elements?

(b) Prove that G must be abelian.

When typing this problem up, write it up carefully: Restate the problem. Write in complete sentences.