Math 3110-101 # Homework #4 **Due: Fri., Feb. 26^{th}, 2021**

Please remember when submitting any work via email or in person to...

PUT YOUR NAME ON YOUR WORK!

#1 The Matrix problem

(a) Compute $A^{-1}B^2$ where $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \in \mathrm{GL}_2(\mathbb{Z}_9)$

(b) Find the cyclic subgroup generated by A. What is the order of A?

#2 Orders of elements and number of such elements.

(a) Make a table which lists the possible orders of elements of $\mathbb{Z}_{495}$. List of the number such elements in the second row.

Order =	1	???	$\cdots$
Number of such elements =	1	???	$\cdots$

How many generators does $\mathbb{Z}_{495}$ have?

(b) Repeat part (a) for D_{44}

Order =	1	???	$\cdots$
Number of such elements =	1	???	$\cdots$

Does D_{44} have a generator? What is/are they? or Why not?

(c) How many elements of order 12 are there in $\mathbb{Z}_{29616}$? What is/are they? or Why are there none?

(d) How many elements of order 9 are there in $\mathbb{Z}_{29616}$? What is/are they? or Why are there none?

#3 Let $g \in G$ (for some group G). Suppose $|g| = 264$. List the *distinct* elements of $\langle g^{220} \rangle$.

#4 Let $x, y \in G$ (for some group G). If there exists some $g \in G$ such $gxg^{-1} = y$, we say x and y are *conjugates*.

(a) Let $y = gxg^{-1}$ for some $g \in G$. Show that $|x| = |y|$ (i.e. conjugates have the same order).

(b) Prove or give a counterexample: $\langle x \rangle = \langle gxg^{-1} \rangle$ (where $x, g \in G$).
In other words, is it true or not that conjugates generate the same cyclic subgroup?