Homework #7

Due: Mon., Mar. 22nd, 2021

Please remember when submitting any work via email or in person to...

PUT YOUR NAME ON YOUR WORK!

- #1 Let H and K be subgroups of G.
 - (a) Suppose that H and K are normal subgroups of G. Show that $H \cap K$ is a normal subgroup of G as well. Note: Please include a careful proof that $H \cap K$ is a subgroup – even though we've shown this before.
 - (b) Let |G| = 100, |H| = 50, and |K| = 20. Using Lagrange's Theorem, what are the possible orders of $H \cap K$?
- #2 Let $H = \{1, x^4, y, x^4y\} \subseteq D_8 = \{1, x, \dots, x^7, y, xy, \dots, x^7y\} = \langle x, y \mid x^8 = 1, y^2 = 1, xyxy = 1 \rangle$. It isn't hard to show that H is closed under the operation in D_8 , thus by the finite subgroup test, H is a subgroup of D_8 . Quickly compute $[D_8 : H]$ (i.e. the index of H in D_8). Then find all of the left and right cosets of H in D_8 . Is H a normal subgroup of D_8 ?
- #3 Direct products of cyclic groups.¹
 - (a) Find the order of (5, 6, 44) in $\mathbb{Z}_9 \times \mathbb{Z}_8 \times \mathbb{Z}_{66}$.
 - (b) Explain why $\mathbb{Z}_{50} \cong \mathbb{Z}_2 \times \mathbb{Z}_{25}$ but $\mathbb{Z}_{50} \not\cong \mathbb{Z}_5 \times \mathbb{Z}_{10}$. In addition, list the distinct orders of elements in both $\mathbb{Z}_2 \times \mathbb{Z}_{25}$ and $\mathbb{Z}_5 \times \mathbb{Z}_{10}$ and give an example of an element of each such order.
- #4 Let G and H be groups.
 - (a) Show $\{e\} \times H = \{(e,h) \mid h \in H\}$ is a normal subgroup of $G \times H$ (where e is the identity of G). Note: You need to show that $\{e\} \times H$ is a subgroup AND that it's normal.
 - (b) Show $G \times H \cong H \times G$.

¹Gallian uses $G \oplus H$ for direct products. I will use the more standard $G \times H$ notation.