**Math 3110-101** 

# Homework #1

**Due: Fri., Jan. 21st, 2022**

Please remember when submitting any work via email or in person to...

**PUT YOUR NAME ON YOUR WORK!**

**#1.** Group axiom basics.

(a) Explain why positive rationals with multiplication, $(\mathbb{Q}_{>0}, \cdot)$, is a group, but negative rationals with multiplication $(\mathbb{Q}_{<0}, \cdot)$ is not.

(b) Which group axioms hold and which fail if we consider $(\mathbb{R}_{\neq 0}, \div)$ non-zero reals with division? Give **concrete** counter-examples for axioms that fail.

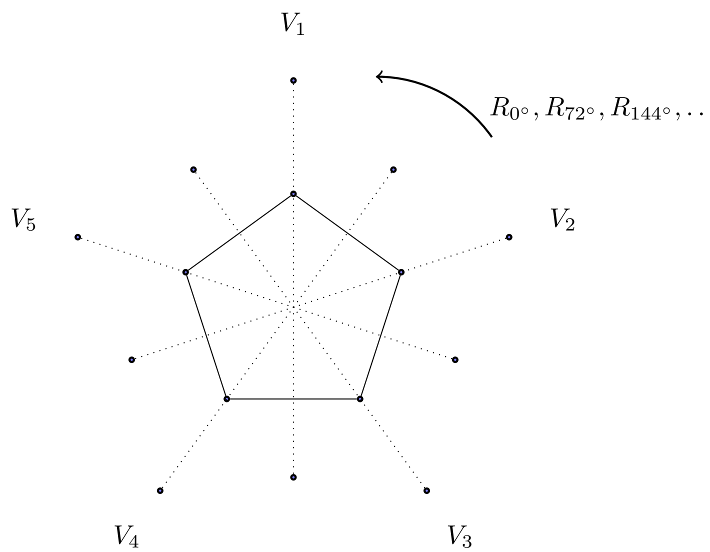
(c) Consider the maximum operation: $x \star y = \max\{x, y\}$. For example, $-8 \star 5 = \max\{-8, 5\} = 5$ since 5 is greater than $-8$. Is $\mathbb{Z}$, the integers, equipped with the max operation a group? Which axioms hold? Give a proof if an axiom holds or a **concrete** counter-example if one fails. Also, is this operation commutative?

**#2.** Let $G$ be a group with identity $e \in G$. Suppose that $g^2 = e$ for all $g \in G$.

(a) What can be said about inverses of elements in $G$? What can be said about orders of elements?

(b) Prove that $G$ must be abelian.

**#3.** Consider the dihedral group $D_5 = \{R_{0^\circ}, R_{72^\circ}, R_{144^\circ}, R_{216^\circ}, R_{288^\circ}, V_1, V_2, V_3, V_4, V_5\}$ (symmetries of a regular pentagon). [Rotations are done counter-clockwise and reflections are labeled in the picture below.]




(a) Compute $V_1 R_{72^\circ}$, $R_{144^\circ} V_3$, and $V_2 V_5$. [Draw some pictures!]

(b) Is $D_5$ Abelian? Why or why not?

(c) Make a table of inverses and orders for each element:

| Element: $g =$ | $R_{0^\circ}$ | $R_{72^\circ}$ | $R_{144^\circ}$ | $R_{216^\circ}$ | $R_{288^\circ}$ | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Inverse: $g^{-1} =$ | ??? | $\cdots$ | | | | | | | | |
| Order: $\|g\| =$ | ??? | $\cdots$ | | | | | | | | |