Homework #2

Due: Fri., Jan. 28th, 2022

Please remember when submitting any work via email or in person to...

PUT YOUR NAME ON YOUR WORK!

#1 Workin' mod 14.

(a) Fill out the following table (if an entry is undefined, write "DNE" = does not exist):

element $x =$	0	1	2	3	4	5	6	7	8	9	10	11	12	13
additive inverse $-x =$	0													
additive order $ x =$	1													
mutiplicative inverse $x^{-1} =$	DNE													
multiplicative order $ x =$	DNE													

Note: Additive inverses and orders go with the group structure of $(\mathbb{Z}_{14}, + \text{mod } 14)$ whereas the multiplicative inverses and orders go with the group structure of $(U(14), \cdot \text{mod } 14)$.

(b) Compute $3^{-5} \cdot (4-12) + 2^{123} \pmod{14}$.

(c) Let
$$A = \begin{bmatrix} 3 & 4 \\ 3 & 1 \end{bmatrix}$$
.

Explain why A does not belong to $GL_2(\mathbb{Z}_6)$.

Then explain why A does belong to $GL_2(\mathbb{Z}_{14})$ and compute A^{-1} .

#2 Use extended Euclidean Algorithm to find 15^{-1} in U(1234).

Show your work. Simplify your answer (your answer should be a number between 0 and 1233).

#3 Let $a, b, n \in \mathbb{Z}$ and n > 1. Suppose that gcd(a, n) = 1 and gcd(b, n) = 1. Show that gcd(ab, n) = 1.

#4 Let $a, b, x, y \in \mathbb{Z}$. Suppose that ax + by = 10.

What can be said about gcd(a, b)?

If gcd(x, y) = 10, what can be said about gcd(a, b)?

RESUBMIT Type up Homework #1 Problem #2 and its solution in LATEX.

Let G be a group with identity $e \in G$. Suppose that $g^2 = e$ for all $g \in G$.

- (a) What can be said about inverses of elements in G? What can be said about orders of elements?
- (b) Prove that G must be abelian.

When typing this problem up, write it up carefully: Restate the problem. Write in complete sentences.