Math 3110 **Due: Mon., Apr. 29th, 2024**

Homework #10
Revision Problem

Name: My Name Goes Here

Revision Problem (Homework #9 Problem #4): Prove that $\mathbb{Q}[\sqrt{-3}] = \{a + b\sqrt{-3} \mid a, b \in \mathbb{Q}\}$ is a field.

Note: Use a subring test to show it is a ring (it is a subring of $\mathbb{C}$).
Then use the "conjugate trick" to show it has multiplicative inverses.

Proof: We will show that $\mathbb{Q}[\sqrt{-3}]$ is a subfield of $\mathbb{C}$ (i.e., the complex numbers).

Non-empty Subset: Clearly $\mathbb{Q}[\sqrt{-3}]$ is a subset of $\mathbb{C}$. Also, $1 = 1 + 0\sqrt{-3} \in \mathbb{Q}[\sqrt{-3}]$.

Closure under stuff: ... Fractions in LaTeX? Try $\frac{abcd}{efgh}$.

Therefore, $\mathbb{Q}[\sqrt{-3}]$ is a sub*field* of $\mathbb{C}$ (and thus it is a field). $\square$