Homework #2

Please remember when submitting any work via email or in person to. . .

> **PUT YOUR NAME ON YOUR WORK!**

#1 Workin' mod 9.

   (a) Create (and fill) a table as follows (if an entry is undefined, write "DNE" = does not exist):

element $x =$	0	1	2	3	4	5	6	7	8
additive inverse $-x =$	0								
additive order $|x| =$	1								
mutiplicative inverse $x^{-1} =$	DNE								
multiplicative order $|x| =$	DNE								

   Note: Additive inverses and orders go with the group structure of $(\mathbb{Z}_9, + \bmod 9)$ whereas the multiplicative inverses and orders go with the group structure of $(U(9), \cdot \bmod 9)$.

   (b) Compute $\quad 3^{1234} + (11 - 33) \cdot 2^{-4} \quad (\bmod 9)$.

   (c) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

   Explain why A does not belong to $\mathrm{GL}_2(\mathbb{Z}_8)$.

   Then explain why A does belong to $\mathrm{GL}_2(\mathbb{Z}_9)$ and compute A^{-1}.

#2 Use the extended Euclidean Algorithm to find 773^{-1} in $U(2332)$.

   Show your work. Simplify your answer (your answer should be a number between 0 and 2331).

#3 Let $a, b, x, y \in \mathbb{Z}$ such that $ax + by = 15$. What can be said about $\gcd(a, b)$?

   Suppose in addition we have $\gcd(x, y) = 15$, then prove a and b are relatively prime.

#4 Let $a, b, c \in \mathbb{Z}$. Assume a divides b and a divides c. Also, suppose b and c are relatively prime (i.e., $\gcd(b, c) = 1$). Show a divides bc.

   Hint: Use an integer linear combination involving b and c then multiply that by bc.

#5 RESUBMIT Type up Homework #1 Problem #2 and its solution in LaTeX.

> Let G be a group with identity $e \in G$.
>
>   (a) Give a concrete example of a group G and elements $a, b \in G$ where $(ab)^2 \neq a^2 b^2$.
>
>   (b) Prove G is an abelian group if and only if for all $a, b \in G$, $(ab)^2 = a^2 b^2$.

When typing this problem up, write it up carefully: Restate the problem. Write in complete sentences.