Homework #3

Please remember when submitting any work via email or in person to...

<div style="border:1px solid red;">

PUT YOUR NAME ON YOUR WORK!

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#1 A function problem. Recall that for a function $h : X \to Y$, if $A \subseteq X$ and $B \subseteq Y$, then...

$$h(A) = \{h(x) \mid x \in A\} \subseteq Y \qquad \text{and} \qquad h^{-1}(B) = \{x \in X \mid h(x) \in B\} \subseteq X$$

(a) Let $f : \mathbb{Z} \to \mathbb{Z}$ be defined by $f(x) = |2 - x| + 2$.

Suggestion: Start off by making a table of values of $f(x)$ for small x's to see what outputs look like.

    i. Explain why f is not 1-1.

    ii. Explain why f is not onto.

    iii. Let $A = \{-2, -1, 0, 1, 2, 3, 4\}$. Find $f(A) = \{f(x) \mid x \in A\}$.

        (This is the image of the set A under the map f.)

    iv. Let $B = \{0, 1, 2, 3, 4, 5\}$. Find $f^{-1}(B) = \{x \in \mathbb{Z} \mid f(x) \in B\}$.

        (This is the inverse image of B under the map f.)

(b) Let $g : X \to Y$.

For $A, B \subseteq X$ show $g(A \cap B) \subseteq g(A) \cap g(B)$.

Then for $C, D \subseteq Y$ show $g^{-1}(C \cup D) = g^{-1}(C) \cup g^{-1}(D)$.

#2 Dihedral groups: generators and relations style. Recall that ...

$$D_n = \langle x, y \mid x^n = 1, \ y^2 = 1, \ \text{and} \ (xy)^2 = 1 \rangle = \{1, x, x^2, \ldots, x^{n-1}, y, xy, x^2y, \ldots, x^{n-1}y\}$$

(a) Write down the Cayley table for D_4.

(b) Fill out the following table for D_8:

element $g =$	1	x	x^2	x^3	x^4	x^5	x^6	x^7	y	xy	x^2y	x^3y	x^4y	x^5y	x^6y	x^7y
inverse $g^{-1} =$	1															
order $|g| =$	1															

(c) Simplify $x^{10}y^{-5}x^{-2}y^{48}x^3$ in D_7.

#3 Let $H = \left\{ \begin{bmatrix} 1 & 0 \\ 3a & 1 \end{bmatrix} \ \middle| \ a \in \mathbb{Z} \right\}$. Show H is a subgroup of $\text{GL}_2(\mathbb{Z})$.

#4 Let G be an abelian group and $K = \{g \in G \mid \text{the order of } g \text{ divides } 2\}$.

(a) Show K is a subgroup of G.

(b) Give an example of a *non-abelian* group G where K (defined the same way) is not a subgroup.

#5 RESUBMIT Type up Homework #2 Problem #4 and its solution in L^AT_EX.

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Let $a, b, c \in \mathbb{Z}$. Assume a divides b, a divides c, and $\gcd(b, c) = 1$ (i.e., b and c are relatively prime).
Show a divides bc.

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When typing this problem up, write it up carefully: Restate the problem. Write in complete sentences.