Math 3110 # Homework #4 **Due: Fri., Feb. 23$^{\text{rd}}$, 2024**

Please remember when submitting any work via email or in person to...

PUT YOUR NAME ON YOUR WORK!

#1 The Matrix... ...problem

Consider $A = \begin{bmatrix} 2 & 0 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ in $\mathbb{Z}_7^{2\times 2}$.

(a) Explain why these matrices belong to $\text{GL}_2(\mathbb{Z}_7)$.

(b) Compute A^2B^{-1}.

(c) Find the cyclic subgroup generated by A (working in $\text{GL}_2(\mathbb{Z}_7)$). What is the order of A?

#2 Orders of elements and number of such elements.

Note: It might be helpful to obtain a list of divisors of various numbers appearing here (like 350 and 98). I recommend using a tool like the website WolframAlpha.com. In Wolfram Alpha, you could type something like "divisors of 350" and it will give you a list.

(a) Make a table which lists the possible orders of elements of $\mathbb{Z}_{350}$. List of the number such elements in the second row.

Order =	1	???	···
Number of such elements =	1	???	···

How many generators does $\mathbb{Z}_{350}$ have?

(b) Repeat part (a) for D_{98}

Order =	1	???	···
Number of such elements =	1	???	···

Does D_{98} have a generator? What is/are they? or Why not?

(c) How many elements of order 18 are there in $\mathbb{Z}_{12345678}$? What is/are they? or Why are there none?

(d) How many elements of order 18 are there in $\mathbb{Z}_{123456789}$? What is/are they? or Why are there none?

#3 Let $g \in G$ (for some group G). Suppose $|g| = 728$. List the *distinct* elements of $\langle g^{455} \rangle$.

#4 Let $x, y \in G$ (for some group G). If there exists some $g \in G$ such $gxg^{-1} = y$, we say x and y are *conjugates*.

(a) Show the relation: $x \sim y$ iff y is a conjugate of x (i.e., there is some $g \in G$ such that $gxg^{-1} = y$) is an *equivalence relation*.

In other words, show $\sim$ is reflexive: for all $x \in G$, we have $x \sim x$; $\sim$ is symmetric: if $x \sim y$, then $y \sim x$; and $\sim$ is transitive: if $x \sim y$ and $y \sim z$, then $x \sim z$.

(b) Since conjugacy is an equivalence relation, this partitions G into equivalence classes:

$$[x] = \{y \in G \mid y \text{ is a conjugate of } x\}$$

Suppose G is abelian and $x \in G$. What is $[x]$?

(c) Find the conjugacy classes of $D_4 = \langle x, y \mid x^4 = 1, y^2 = 1, (xy)^2 = 1 \rangle = \{1, x, x^2, x^3, y, xy, x^2y, x^3y\}$.