Math 3110 # Homework #5 **Due: Fri., Mar. 1st, 2024**

Please remember when submitting any work via email or in person to...

PUT YOUR NAME ON YOUR WORK!

#1 Let $\sigma = (12)(456)$. Consider the permutations τ in the table below.

$\Longrightarrow$ Remember to give *simplified good manners* answers!

Fill out the following table:

$\tau =$	$(12)(345)(246)(13)$	$(146)(35)(2534)$	$(245)(1234)$
τ simplified (as disjoint cycles):			
The order of τ: $\|\tau\| =$			
The inverse of τ: $\tau^{-1} =$			
τ as a product of transpositions:			
τ conjugated by σ: $\sigma\tau\sigma^{-1} =$			
A power of τ: $\tau^{33} =$			

#2 Orders in S_n.

(a) What are the orders of the elements in S_8? Give an example of an element with each order.

Note: Recall that orders of permutations depend on their cycle structure and that cycle structures correspond to partitions. There are 22 partitions of 8. Typing "partitions of 8" into Wolfram Alpha (and asking it to show "More") you can get a complete list. That said, many partitions yield redundant element orders. You don't need to catalog of all possible structures. Just give an example for each *distinct* element order. [For example: (12) has order 2, so examples like (12)(34) and (12)(34)(56) – also order 2 – are redundant.]

(b) A 60 cycle in S_{60} has order 60. But elements of order 60 show up much sooner than that. What is the smallest n such that S_n has an element of order $60 = 2^2 \cdot 3 \cdot 5$? Give an example of such an element.

(c) When does order 9 first show up in a permutation group S_n? Give an example of such an element and explain why 9 does not show up sooner.

#3 RESUBMIT Type up Homework #4 Problem #4 part (a) and its solution in LaTeX.

> Show the relation: $x \sim y$ iff y is a conjugate of x (i.e., there exists $g \in G$ such that $gxg^{-1} = y$) is an *equivalence relation.*
>
> In other words, show...
>
> - $\sim$ is reflexive: for all $x \in G$, we have $x \sim x$;
> - $\sim$ is symmetric: if $x \sim y$, then $y \sim x$; and
> - $\sim$ is transitive: if $x \sim y$ and $y \sim z$, then $x \sim z$.

When typing this problem up, please write carefully: Restate the problem. Write in complete sentences.