Math 3110 # Homework #6 **Due:** **Wed., Mar. 6th, 2024**

Please remember when submitting any work via email or in person to...

PUT YOUR NAME ON YOUR WORK!

#1 The following pairs of groups are **not** isomorphic. Prove it.

Hint/Note: Find a (group) property that does not match.

(a) $\mathbb{R}^{2\times 2}$ and $\mathrm{GL}_2(\mathbb{R})$

(b) $U(25)$ and D_{10}

(c) A_5 and D_{30}

(d) $\mathbb{R}_{\neq 0}$ and $\mathbb{C}_{\neq 0}$ (non-zero reals and complex numbers both under multiplication)

#2 The following pairs of groups are isomorphic. Prove it.

(a) $U(14)$ and $\mathbb{Z}_6$

Hint: You could cite a theorem and prove this in one line.

(b) $G = \left\langle \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \right\rangle = \left\{ \begin{bmatrix} a & 2a \\ 0 & 3a \end{bmatrix} \;\middle|\; a \in \mathbb{Z} \right\}$ is a (cyclic) subgroup of $\mathbb{Z}^{2\times 2}$ (2×2 matrices with integer entries are a group under matrix addition). Show $G \cong \mathbb{Z}$.

~~*Note:* You need to come up with an isomorphism, $\varphi : G \to \mathbb{Z}$, and prove it is an isomorphism.~~

#3 Cayley's theorem tells us that $D_3 = \langle x, y \mid x^3 = 1, y^2 = 1, (xy)^2 = 1 \rangle = \{1, x, x^2, y, xy, x^2y\}$ is isomorphic to a subgroup of S_6 (since $|D_3| = 2 \cdot 3 = 6$. Find such a subgroup using the ordering of elements: $1, x, x^2, y, xy, x^2y$ (so, for example, x^2 is element #3). To help you get started, left multiplication by x is $1 \mapsto x$, $x \mapsto x^2$, $x^2 \mapsto 1$, $y \mapsto xy$, $xy \mapsto x^2y$, and $x^2y \mapsto y$ which means #1 $\mapsto$ #2, #2 $\mapsto$ #3, etc. This is the permutation $(123)(456)$. So $D_3 \cong \{(1), (123)(456), ???\}$.