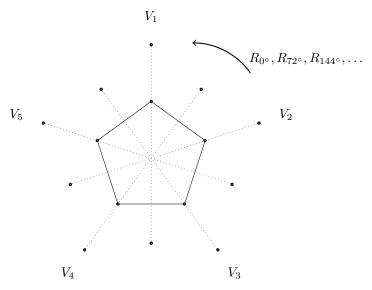
## Homework #1

Due: Fri., Jan. 24<sup>th</sup>, 2025

Please remember when submitting any work via email or in person to...

## PUT YOUR NAME ON YOUR WORK!

- **#1.** Group axiom basics.
  - (a) Explain why positive rationals with multiplication,  $(\mathbb{Q}_{>0}, \cdot)$ , is a group, but negative rationals with multiplication  $(\mathbb{Q}_{<0}, \cdot)$  is not.
  - (b) Which group axioms hold and which fail if we consider  $(\mathbb{R}_{\neq 0}, \div)$  non-zero reals with division? Give **concrete** counter-examples for axioms that fail.
  - (c) Consider the maximum operation:  $x \star y = \max\{x, y\}$ . For example,  $-8 \star 5 = \max\{-8, 5\} = 5$  since 5 is greater than -8. Is  $\mathbb{Z}$ , the integers, equipped with the max operation a group? Which axioms hold? Give a proof if an axiom holds or a **concrete** counter-example if one fails. Also, is this operation commutative?
- #2. Let G be a group with identity  $e \in G$ . Suppose that  $g^2 = e$  for all  $g \in G$ .
  - (a) What can be said about inverses of elements in G? What can be said about orders of elements?
  - (b) Prove that G must be abelian.
- #3. Consider the dihedral group  $D_5 = \{R_{0^{\circ}}, R_{72^{\circ}}, R_{144^{\circ}}, R_{216^{\circ}}, R_{288^{\circ}}, V_1, V_2, V_3, V_4, V_5\}$  (symmetries of a regular pentagon). [Rotations are done counter-clockwise and reflections are labeled in the picture below.]



- (a) Compute  $V_1R_{72^{\circ}}$ ,  $R_{144^{\circ}}V_3$ , and  $V_2V_5$ . [Draw some pictures!]
- (b) Is  $D_5$  Abelian? Why or why not?
- (c) Make a table of inverses and orders for each element:

Element:	g =	$R_{0^{\circ}}$	$R_{72^{\circ}}$	$R_{144^{\circ}}$	$R_{216^{\circ}}$	$R_{288^{\circ}}$	$V_1$	$V_2$	$V_3$	$V_4$	$V_5$
Inverse:	$g^{-1} =$	???									
Order:	g  =	???									