Math 3110 **Homework #10** **Due: Mon., Apr. 28$^{\text{th}}$, 2025**

Revision Problem

[Type up Homework #9 Problem #4 and its solution in LaTeX.]

Name: **My Name Goes Here**

Proposition: Equipped with the usual operations, $\mathbb{Q}[\sqrt{5}] = \{a + b\sqrt{5} \mid a, b \in \mathbb{Q}\}$ is a field.

Notes: Use a subring (of $\mathbb{R}$) test to show it is a ring. Use a "conjugate trick" to compute multiplicative inverses. Don't forget to mention the extra "trivial" bits about being a field: It is a *commutative* ring with *unity*. Specifically note $1 = 1 + 0\sqrt{5} \in \mathbb{Q}[\sqrt{5}]$ (and of course $1 \neq 0$).

Proof: First, we prove that $\mathbb{Q}[\sqrt{5}]$ is a subring of $\mathbb{R}$.

- Maybe this step is obvious.
- More...
- Possibly more...

Therefore, by the subring test, $\mathbb{Q}[\sqrt{5}]$ is a subring of $\mathbb{R}$. In addition, we notice STUFF, so that $\mathbb{Q}[\sqrt{5}]$ is STUFF. Finally, we check .....

- Yet another step....

Therefore, $\mathbb{Q}[\sqrt{5}]$ is a subfield of $\mathbb{R}$. In particular, $\mathbb{Q}[\sqrt{5}]$ is a field. $\square$