Math 3110 # Homework #3 **Due: Fri., Feb. 7th, 2025**

Please remember when submitting any work via email or in person to...

PUT YOUR NAME ON YOUR WORK!

#1 A function problem

(a) Let $f : \mathbb{Z} \to \mathbb{Z}$ be defined by $f(x) = |2x+1| + 3$.

i. Show f is not 1-1.

ii. Show f is not onto.

iii. Let $A = \{-3, -2, -1, 0, 1, 2, 3\}$. Find $f(A) = \{f(x) \mid x \in A\}$ (the image of the set A under the map f).

iv. Let $B = \{0, 1, 2, 3, 4, 5, 6\}$. Find $f^{-1}(B) = \{x \in \mathbb{Z} \mid f(x) \in B\}$ (the inverse image of B).

(b) Let $g : X \to Y$. Prove that g is onto if and only if $g^{-1}(B) \neq \phi$ (the inverse image of B is non-empty) for all non-empty subsets of Y: $\phi \neq B \subseteq Y$.

Recall that for $A \subseteq X$ and $B \subseteq Y$ ...

$$f(A) = \{f(x) \mid x \in A\} \subseteq Y \qquad \text{and} \qquad f^{-1}(B) = \{x \in X \mid f(x) \in B\} \subseteq X$$

#2 Dihedral groups: generators and relations style. Recall that ...

$$D_n = \langle x, y \mid x^n = 1,\ y^2 = 1,\ \text{and } (xy)^2 = 1 \rangle = \{1, x, x^2, \dots, x^{n-1}, y, xy, x^2y, \dots, x^{n-1}y\}$$

(a) Write down the Cayley table for D_4. Determine the contents of the center of D_4 (i.e., $Z(D_4) = \{???\}$).
Note: Recall that $Z(G) = \{a \in G \mid \text{for all } g \in G,\ ag = ga\}$ is the *center* of a group G.

(b) Fill out the following table for D_6:

element $g =$	1	x	x^2	x^3	x^4	x^5	y	xy	x^2y	x^3y	x^4y	x^5y
inverse $g^{-1} =$	1											
order $\lvert g\rvert =$	1											

(c) Simplify $yx^7y^{-3}x^{11}y^8x^{-2}$ in D_6.

#3 Let $H = \left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \in \mathbb{Z}^{2\times 2} \;\middle|\; a+b+c = 0 \right\}$. Show H is a subgroup of $\mathbb{Z}^{2\times 2}$.
Note: $\mathbb{Z}^{2\times 2}$ is an Abelian group under matrix *addition*.

#4 Let H and K be subgroups of a group G. Show that $H \cap K$ (the intersection of H and K) is a subgroup of G.

RESUBMIT Type up Homework #2 Problem #3 and its solution in LaTeX.

Let $a, b, n \in \mathbb{Z}$ and $n > 1$. Suppose that $\gcd(a,n) = 1$ and $\gcd(b,n) = 1$. Show that $\gcd(ab,n) = 1$.

When typing this problem up, write it up carefully: Restate the problem. Write in complete sentences.