Math 3110

Homework #4

Please remember when submitting any work via email or in person to...

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PUT YOUR NAME ON YOUR WORK!
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#1 The Matrix

(a) Compute
$$A^{-1}B^2$$
 where $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \in \operatorname{GL}_2(\mathbb{Z}_9)$

(b) Find the cyclic subgroup generated by A. What is the order of A?

#2 Orders of elements and number of such elements.

problem

(a) Make a table which lists the possible orders of elements of \mathbb{Z}_{495} . List of the number such elements in the second row.

Order =	1	???	•••
Number of such elements $=$	1	???	

How many generators does \mathbb{Z}_{495} have?

(b) Repeat part (a) for D_{44}

Order =	1	???	•••
Number of such elements $=$	1	???	•••

Does D_{44} have a generator? What is/are they? or Why not?

- (c) How many elements of order 12 are there in \mathbb{Z}_{29616} ? What is/are they? or Why are there none?
- (d) How many elements of order 9 are there in \mathbb{Z}_{29616} ? What is/are they? or Why are there none?

#3 Let $g \in G$ (for some group G). Suppose |g| = 264. List the distinct elements of $\langle g^{220} \rangle$.

#4 Let $x, y \in G$ (for some group G). If there exists some $g \in G$ such $gxg^{-1} = y$, we say x and y are conjugates.

- (a) Let $y = gxg^{-1}$ for some $g \in G$. Show that |x| = |y| (i.e. conjugates have the same order).
- (b) Prove or give a counterexample: $\langle x \rangle = \langle gxg^{-1} \rangle$ (where $x, g \in G$). In other words, is it true or not that conjugates generate the same cyclic subgroup?