Math 3110 | # Homework #5 | **Due: Fri., Feb. 28$^{\text{th}}$, 2025**

Revision Problem

[Type up Homework #4 Problems #4 and its solution in LaTeX.]

Name: **My Name Goes Here**

Let $x, y \in G$ (for some group G). If there exists some $g \in G$ such $gxg^{-1} = y$, we say x and y are *conjugates*.

(a) Let $y = gxg^{-1}$ for some $g \in G$. Show that $|x| = |y|$ (i.e. conjugates have the same order).

First, we must prove a lemma:

Lemma: Let $x, g \in G$ (for some group G) and k is a non-negative integer. Then $(gxg^{-1})^k = gx^kg^{-1}$.

Proof: We proceed by induction.

- First, we show our base case holds. When $k = 0$, BASE CASE HERE.
- Next, we make the inductive assumption that $(gxg^{-1})^k = gx^kg^{-1}$ holds for some non-negative integer k. Then STUFF GOES HERE showing this holds for $k + 1$.

Therefore, by induction $(gxg^{-1})^k = gx^kg^{-1}$ holds for every non-negative integer k. □

Proposition: Let $y = gxg^{-1}$ for some $g \in G$. Show that $|x| = |y|$ (i.e. conjugates have the same order).

Proof: MY PROOF GOES HERE. □

(b) Prove or give a counterexample: $\langle x \rangle = \langle gxg^{-1} \rangle$ (where $x, g \in G$).

In other words, is it true or not that conjugates generate the same cyclic subgroup?

Consider BLAH BLAH. Therefore, BLAH.