Math 3110

Homework #5

Please remember when submitting any work via email or in person to...

PUT YOUR NAME ON YOUR WORK!

#1 Let $\sigma = (123)(45)$. Fill out the following table:

$\tau =$	(1432)(56)(254)	(1234)(1423)(246)	(12)(345)(1357)
τ simplified (as disjoint cycles):			
The order of τ : $ \tau =$			
The inverse of τ : $\tau^{-1} =$			
τ as a product of transpositions:			
τ conjugated by σ : $\sigma \tau \sigma^{-1} =$			
A power of τ : $\tau^{99} =$			

#2 Orders in S_n .

(a) What are the orders of the elements in S_7 ? Give an example of an element with each order.

- (b) Does S_{10} have an element of order 30? If so, find one. If not, explain why not.
- (c) Does S_{10} have an element of order 25? If so, find one. If not, explain why not.

RESUBMIT Type up Homework #4 Problems #4 and its solution in LATEX.

Let $x, y \in G$ (for some group G). If there exists some $g \in G$ such $gxg^{-1} = y$, we say x and y are *conjugates*.

- (a) Let $y = gxg^{-1}$ for some $g \in G$. Show that |x| = |y| (i.e. conjugates have the same order). **Note:** You will need to know that $(gxg^{-1})^k = gx^kg^{-1}$ for every non-negative integer k. I let you get away with informal justifications on original submissions. This time I would like you to include a careful
- proof of this fact (use induction on k).
 (b) Prove or give a counterexample: ⟨x⟩ = ⟨gxg⁻¹⟩ (where x, g ∈ G).
 In other words, is it true or not that conjugates generate the same cyclic subgroup?

When typing this problem up, please write carefully: Restate the problem. Write in complete sentences.