Math 3110 # Homework #6 **Due: Wed., Mar. 5th, 2025**

Please remember when submitting any work via email or in person to...

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#1 Cayley's theorem tells us that $Q = \{\pm 1, \pm i, \pm j, \pm k\}$ is isomorphic to a subgroup of S_8. Find such a subgroup using the ordering of elements: $1, -1, i, -i, j, -j, k, -k$ (so, for example, i is element #3). To help you get started, left multiplication by -1 sends 1 to -1, -1 to 1, i to $-i$, etc. so it sends 1 to 2, 2 to 1, 3 to 4, etc. Thus -1 corresponds with (12)(34)(56)(78).

#2 The following pairs of groups are isomorphic. Prove it.

(a) $U(7)$ and $\mathbb{Z}_6$
Hint: You could cite a theorem and prove this in one line.

(b) $H = \left\{ \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} \;\middle|\; n \in \mathbb{Q} \right\}$ and $\mathbb{Q}$
Note: You need to come up with an isomorphism, $\varphi : H \to \mathbb{Q}$, and prove it is an isomorphism.

#3 The following pairs of groups are **not** isomorphic. Prove it.
Hint/Note: Find a (group) property that does not match.

(a) $\mathbb{Z}_{246}$ and D_{123}

(b) $(\mathbb{Z}_{99})^{2\times 2}$ and $\mathrm{GL}_2(\mathbb{Z})$

(c) A_4 and D_6

(d) $\mathbb{R}_{\neq 0}$ and $\mathbb{C}_{\neq 0}$ (non-zero reals and complex numbers both under multiplication)

#4 Let $\varphi : G_1 \to G_2$ be an isomorphism

(a) Show $\varphi(\langle g \rangle) = \langle \varphi(g) \rangle$ for any $g \in G_1$. [This implies that G_1 is cyclic iff G_2 is cyclic.]
(b) Let K be a subgroup of G_2. Show that $\varphi^{-1}(K) = \{g \in G_1 \mid \varphi(g) \in K\}$ is a subgroup of G_1.