Math 3130 **Due: June 25th, 2019**

Homework #4

Be sure to show your work!

You may use technology to complete this assignment. Although much of it *could* be done by hand.

1. Solve the initial value problems.
[Do this "by hand" using the method of undetermined coefficients. Technology can help confirm answers.]

(a) $y'' + 9y = 6\sin(t)$ with $y(0) = 1$ and $y'(0) = -1$.

(b) $y'' + 9y = 6\sin(3t)$ with $y(0) = 1$ and $y'(0) = -1$.

(c) Graph both solutions. Even though the IVPs are very similar, the solutions look very different. Why?

2. Find the general solution of $y'' + 3y' + y = 5\cos(3t)$. Write your solution in real form (no complex stuff). In addition, the forced response (the particular solution coming from the forcing function) is a combination of $\sin(3t)$ and $\cos(3t)$. Rewrite that part of the solution in the form: $A\cos(3t + \phi)$. Your A should be exact. Round your ϕ to 3 decimal places.

3. Consider the system $x' = x(2 - x - y)$ and $y' = y(y - x^2)$.

(a) Find the equilibria for this system. Use the method of almost linear systems (i.e., linearization) to analyze these equilibria. Notice that $(x, y) = (0, 0)$ is one of the equilibria. The linearization technique doesn't help here. Why?

(b) Plot a phase portrait of this system (pick a domain big enough to display all equilibria).

(c) Sketch nullclines (with arrows indicating direction as done in Section 5.2).