Math 3130

Resonance Example

We consider y'' + 4y = cos(omega t) where omega is a parameter.

```
> with(DEtools):
> DEq := diff(y(t),t,t)+4*y(t)=cos(omega*t);
```

$$DEq := \frac{d^2}{dt^2}\, y(t) + 4\, y(t) = \cos(\omega\, t) \tag{1}$$

Typical (i.e., omega is not +2 or -2) solutions look like...

```
> dsolve(DEq);
```

$$y(t) = \sin(2\,t)\,\_C2 + \cos(2\,t)\,\_C1 - \frac{\cos(\omega\,t)}{\omega^2 - 4} \tag{2}$$

Start off with y(0) = y'(0) = 0 and see what gets forced...

```
> dsolve({DEq,y(0)=0,D(y)(0)=0});
```

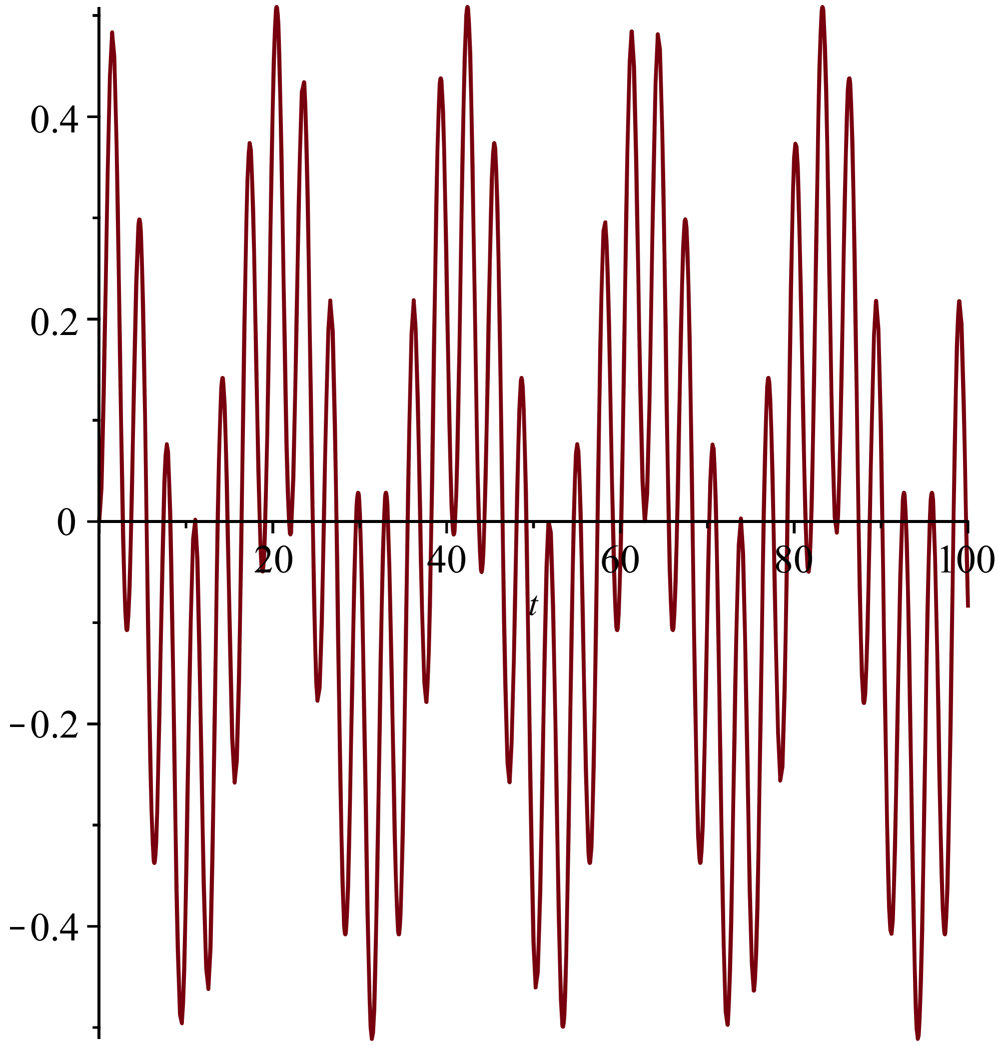
$$y(t) = \frac{\cos(2\,t)}{\omega^2 - 4} - \frac{\cos(\omega\,t)}{\omega^2 - 4} \tag{3}$$

```
> soln := (omega,t) -> cos(2*t)/(omega^2-4)-cos(omega*t)/(omega^2-4):
  'soln(omega,t)' = soln(omega,t);
```

$$soln(\omega, t) = \frac{\cos(2\,t)}{\omega^2 - 4} - \frac{\cos(\omega\,t)}{\omega^2 - 4} \tag{4}$$

omega = 0.3 yields...

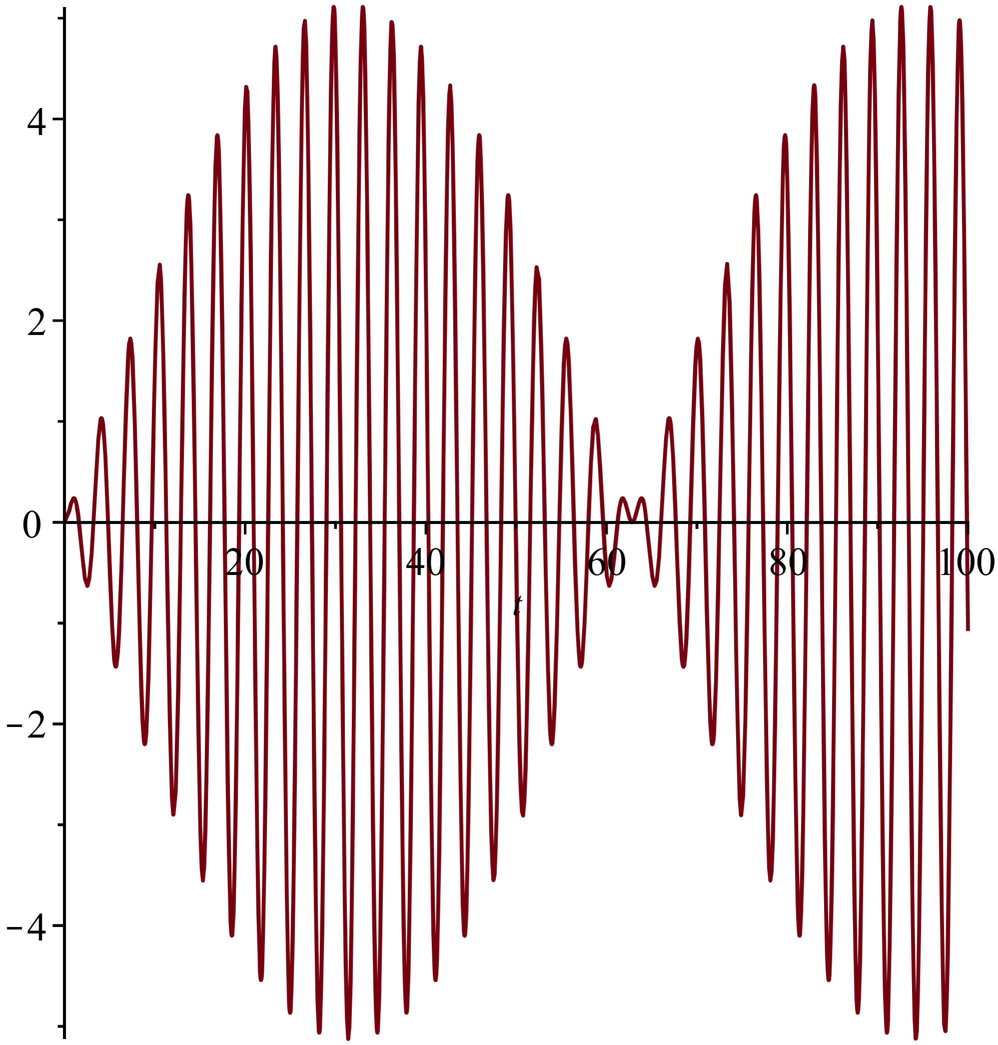
```
> plot(soln(0.3,t),t=0..100);
```



omega = 1.9 (we are getting closer to resonance) yields...

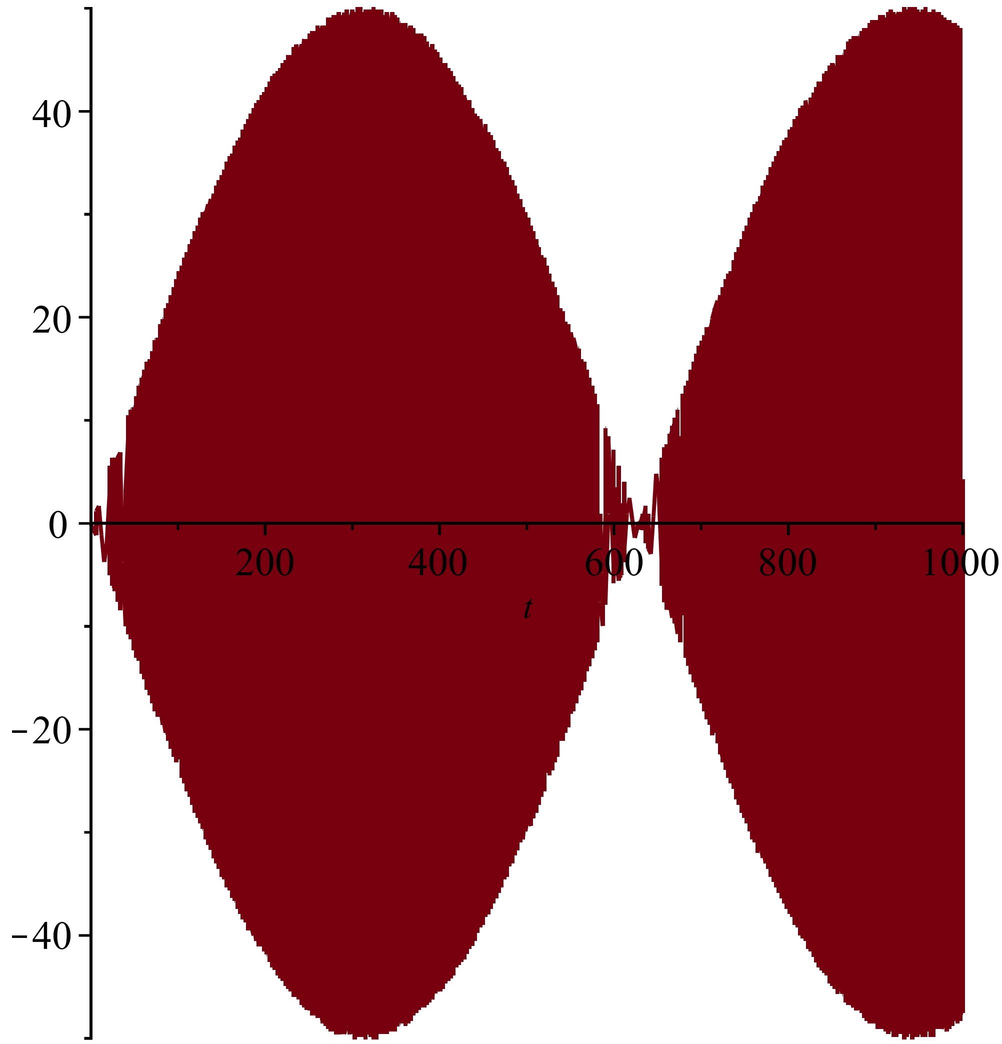
[Notice the "beating"]

```
> plot(soln(1.9,t),t=0..100);
```



omega = 1.99

```
> plot(soln(1.99,t),t=0..1000);
```



At omega = 2 we experience resonance. Here the homogeneous equation's solution contains the forcing term. When we solve, we still
get a linear combination of sin(2t) and cos(2t) but now the forced response is "0.25 t sin(2t)". The amplitude is now unbounded!

```
> subs(omega=2,DEq);
```

$$\frac{d^2}{dt^2} y(t) + 4 y(t) = \cos(2 t) \quad \textbf{(5)}$$

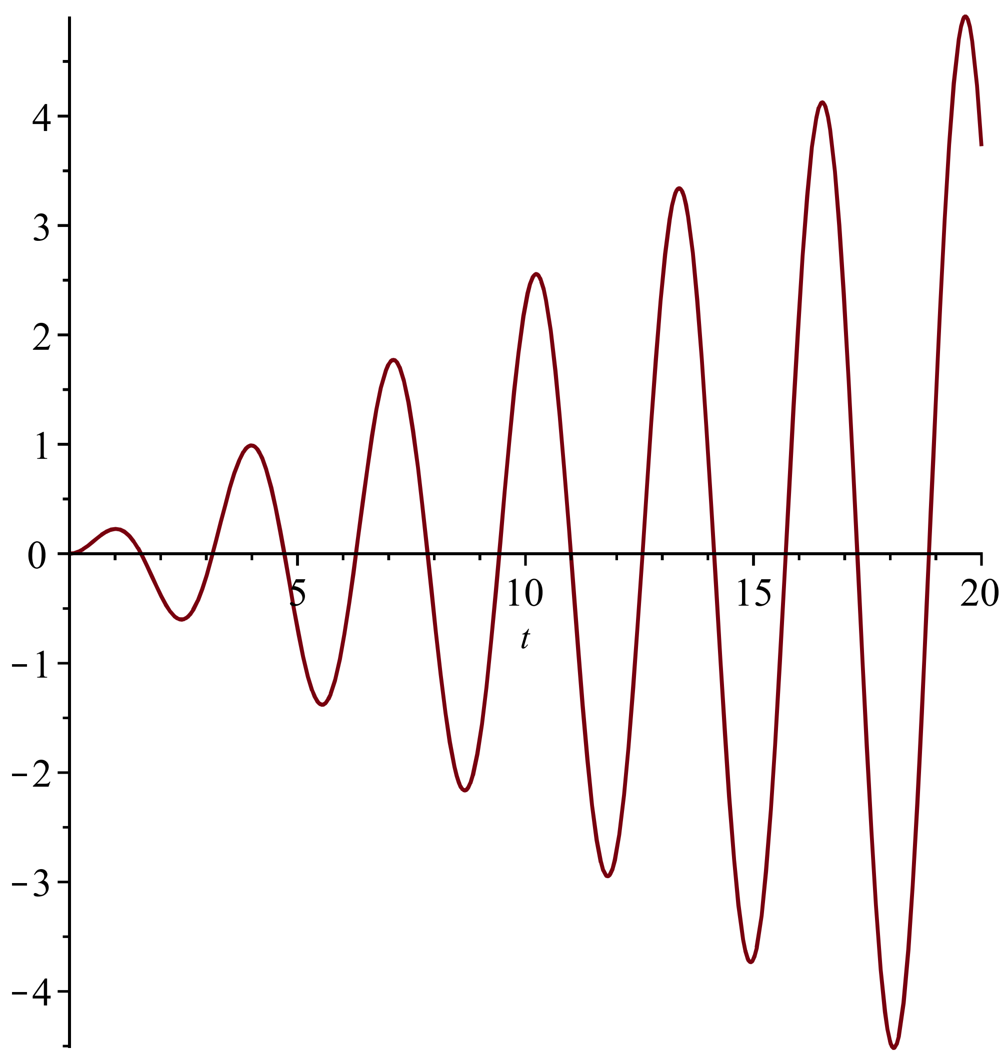
```
> dsolve(subs(omega=2,DEq));
```

$$y(t) = \sin(2 t) \_C2 + \cos(2 t) \_C1 + \frac{\cos(2 t)}{8} + \frac{\sin(2 t) t}{4} \quad \textbf{(6)}$$

```
> dsolve({subs(omega=2,DEq),y(0)=0,D(y)(0)=0});
```

$$y(t) = \frac{\sin(2 t) t}{4} \quad \textbf{(7)}$$

```
> plot((1/4)*sin(2*t)*t,t=0..20);
```



4
3
2
1
0
−1
−2
−3
−4
5
10
15
20
t