Math 3130 - Introduction to Differential Equations

Vector Field Plot and Phase Portrait Examples

```
> restart;
  with(plots):
  with(DEtools):
```

Consider the (predator prey model) 1st order autonomous system: R' = 2R - RF and F' = -3F+RF.

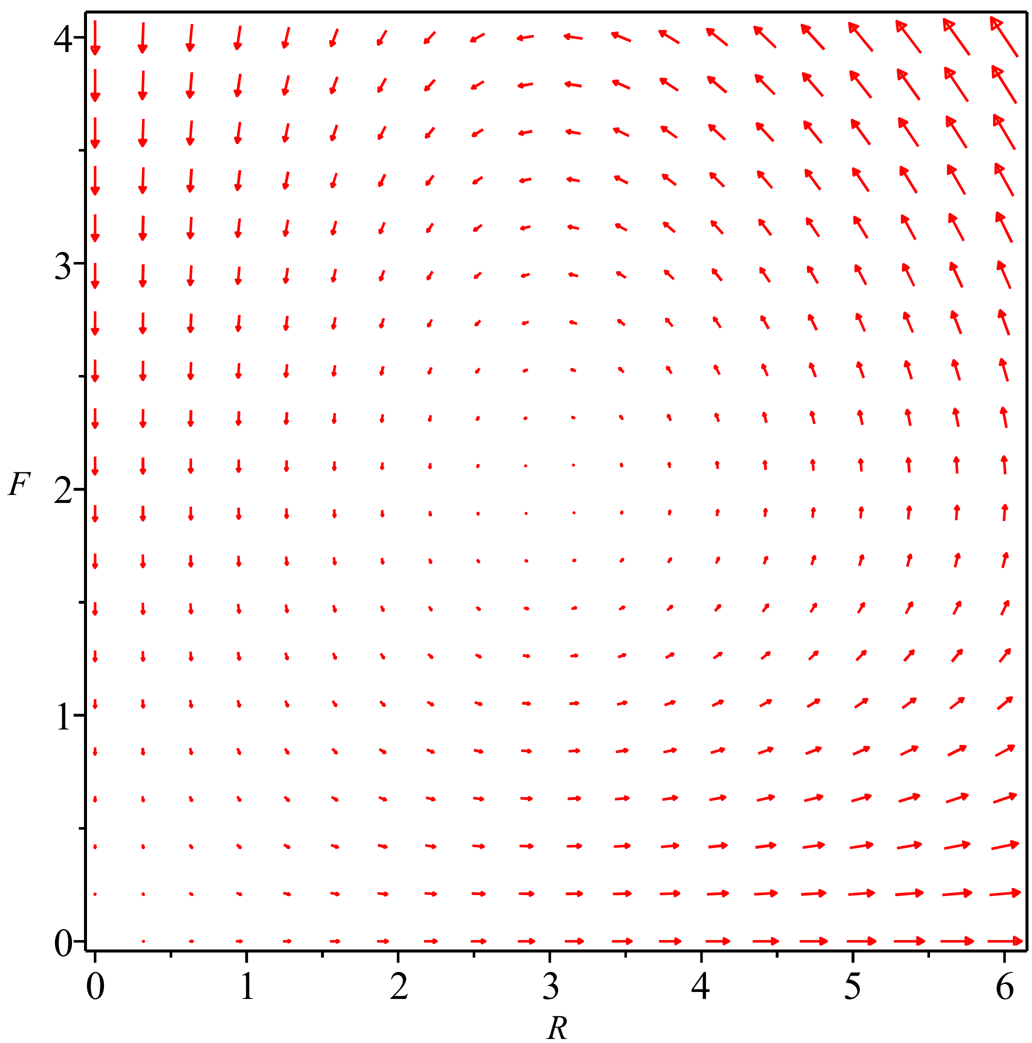
We have the corresponding vector field: V = [2R-RF, -3F+RF]

This plot indicates how the population of rabbits (R(t)) and foxes (F(t)) change relative to each other.

```
> V := [2*R-R*F,-3*F+R*F];
```

$$V := [-R\,F + 2\,R, R\,F - 3\,F] \quad (1)$$

```
> fieldplot(V,R=0..6,F=0..4,arrows=SLIM,color=red,axes=boxed);
```



Now let's give our system a name "DEsys" and then produce a direction field (i.e., vector field without relative scaling) along
with several sample solutions. We choose initial conditions: R(0)=1 & F(0)=1, R(0)=4 & F(0)=4, R(0)=3 & F(0)=1, and R(0)=3 & F(0)=2.8.

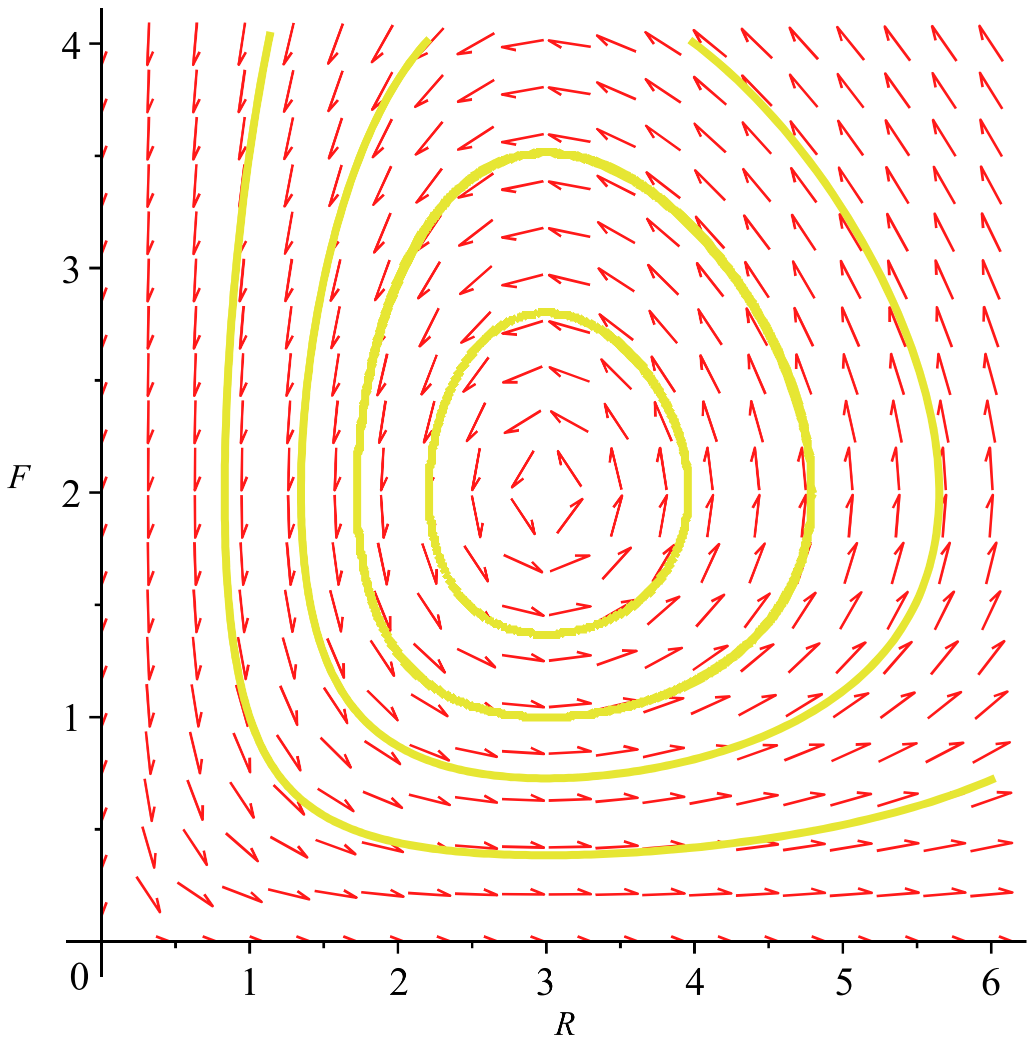
Maple then plots (approximations of) these solutions in the phase plane (RF-plane) given ind. vari. domain -20 <= t <= 20.

The option "numpoints=5000" is to make Maple plot extra points to get less rough looking solutions.

```
> DEsys := [diff(R(t),t)=2*R(t)-R(t)*F(t),diff(F(t),t)=-3*F(t)+R(t)*F(t)
  ];
```

$$DEsys := \left[\frac{d}{dt}R(t) = 2R(t) - R(t)F(t), \frac{d}{dt}F(t) = -3F(t) + R(t)F(t)\right] \quad \textbf{(2)}$$

```
> DEplot(DEsys,[R(t),F(t)],t=-20..20,{[0,1,1],[0,4,4],[0,3,1],[0,3,2.8]}
  ,R=0..6,F=0..4,numpoints=5000);
```



Second example, consider the mass spring system with mass m and spring constant k. Then (ignoring friction and other issues) if y is the distance the mass is pulled away from the spring's natural equilibrium, Hooke says that the force is F = -ky. Newton then gives us my'' = -ky.

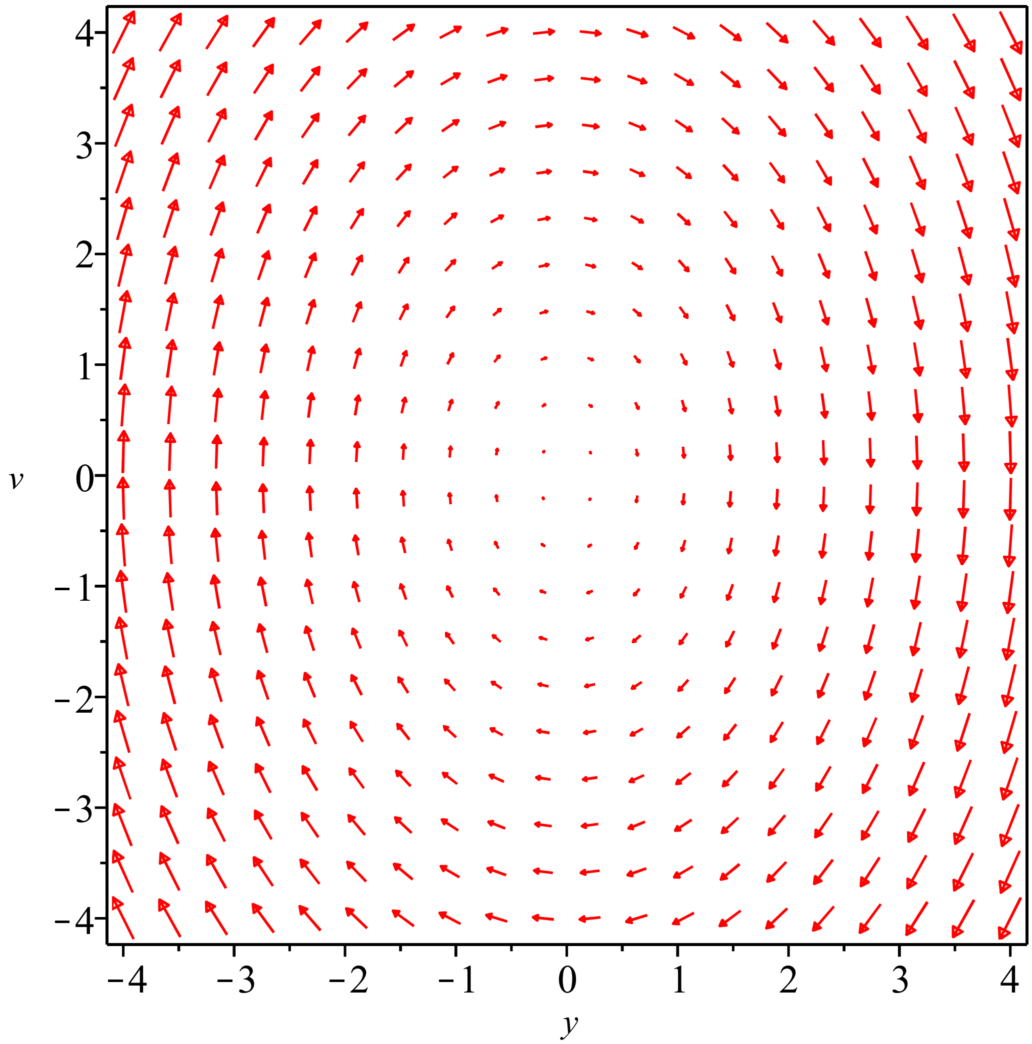
We reduce this second order ODE (which is homogeneous linear with constant coefficients) to a first order system of ODEs (still homogeneous linear with constant coefficients) as follows: let v = y' so that v' = y'' then we have y' = v & v' = -(k/m)y.

For simplicity, suppose k=20 and m=10 so we have y' = v and v' = -2y.

Let's give similar plots as in the last example.

In our phase portrait, we draw solutions with y(0)=1, 2, 3 where v(0)=0. This time the domain 0 <= t <= 10 is enough to see the whole curves.

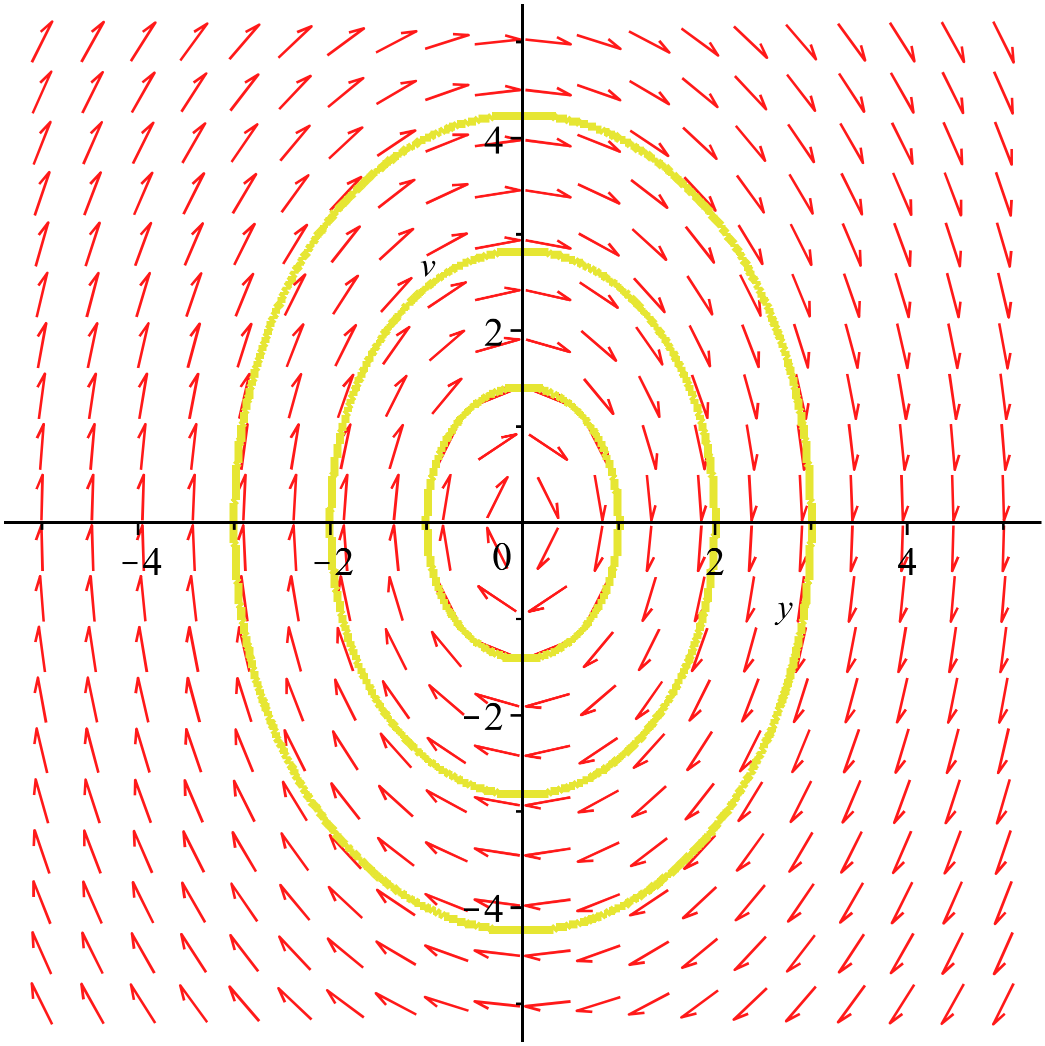
```
> fieldplot([v,-2*y],y=-4..4,v=-4..4,arrows=SLIM,color=red,axes=boxed);
```




```
> DEsys := [diff(y(t),t)=v(t),diff(v(t),t)=-2*y(t)];
```

$$DEsys := \left[\frac{d}{dt}\, y(t) = v(t),\ \frac{d}{dt}\, v(t) = -2\, y(t)\right] \quad (3)$$

```
> DEplot(DEsys,[y(t),v(t)],t=0..10,{[0,1,0],[0,2,0],[0,3,0]},y=-5..5,v=
-5..5,numpoints=2000);
```




```
> soln := dsolve({op(DEsys),y(0)=2,v(0)=0},[y(t),v(t)]);
```

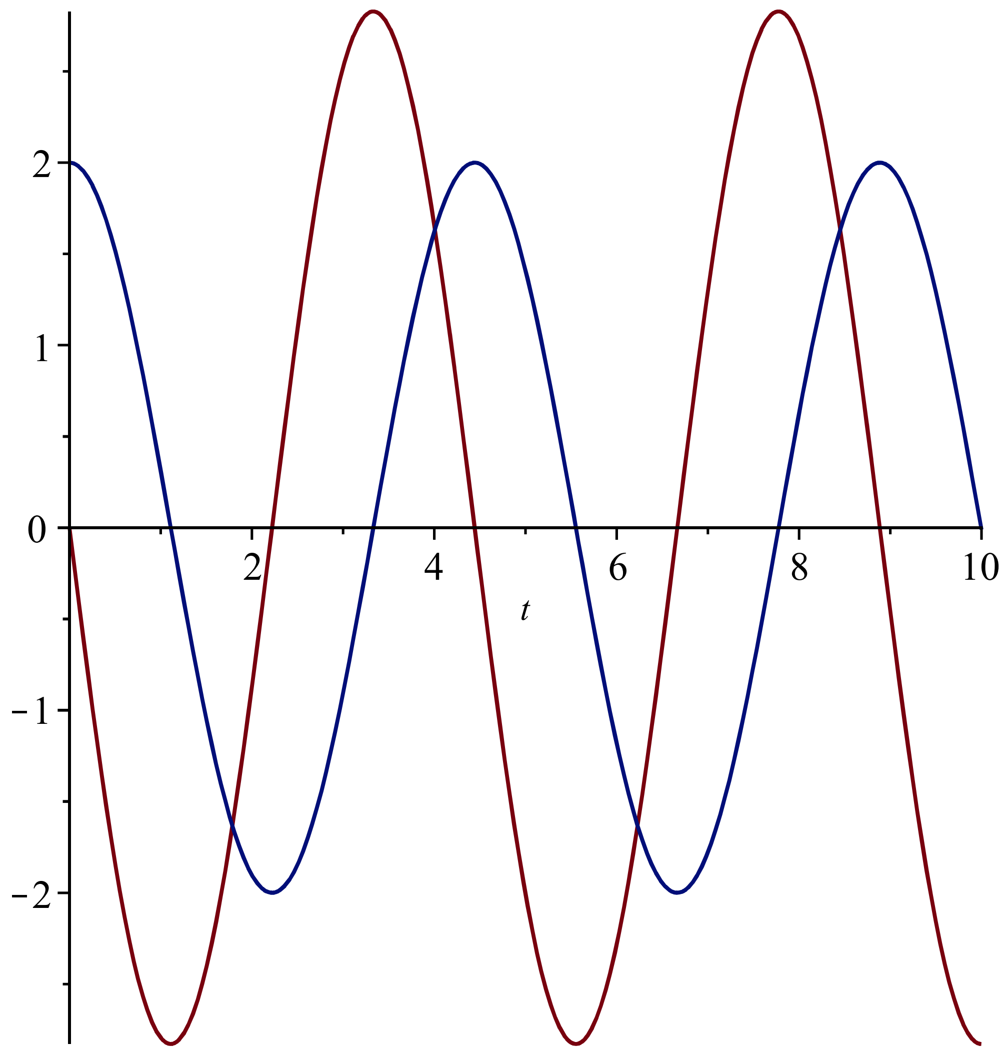
$$soln := \left\{v(t) = -2\sqrt{2}\,\sin\left(\sqrt{2}\,t\right), y(t) = 2\cos\left(\sqrt{2}\,t\right)\right\} \tag{4}$$

```
> vSoln := rhs(soln[1]);
  ySoln := rhs(soln[2]);
```

$$vSoln := -2\sqrt{2}\,\sin\left(\sqrt{2}\,t\right)$$

$$ySoln := 2\cos\left(\sqrt{2}\,t\right) \tag{5}$$

```
> plot([vSoln,ySoln],t=0..10);
```


2
1
0
-1
-2
2
4
6
8
10
t