Math 3220 **Homework #1** **Due: Fri., Jan. 25^{th}, 2019**

#1 (Zorn 1.6 #4) Let $a, b, c, d \in \mathbb{R}$ where $a < b$ and $c < d$. Prove that the intervals $I = (a, b)$ and $J = (c, d)$ have the same cardinality. Do this by finding an appropriate linear function $f : I \to J$ (linear means $f(x) = Ax + B$ for some $A, B \in \mathbb{R}$). Prove that your f is both one-to-one and onto.

#2 Let $A \subseteq B \subseteq \mathbb{R}$. Suppose that $|A| = |B|$ (i.e., A and B have the same cardinality). It does not follow that $A = B$. Give a counterexample to show why. Then, what assumption on B would force this to be true?

#3 (Zorn 1.7 #4) Find all values x that satisfy the given inequality; express answers in interval notation.

(a) $|2x + 7| < 11$

(b) $|2x + 7| > 11$

(c) $|2x^2 + 7| < 11$

#4 Let N be a positive integer and $x_1, \ldots, x_N \in \mathbb{R}$. Show that $|x_1 + \cdots + x_N| \leq |x_1| + \cdots + |x_N|$.

Note: Prove this using induction on N.

#5 Let $a, b \in \mathbb{R}$ where $a < b$. Suppose that $x, y \in (a, b)$. Show that $|x - y| < b - a$.